ASTROPHYSICS Yr 2 2011-2012

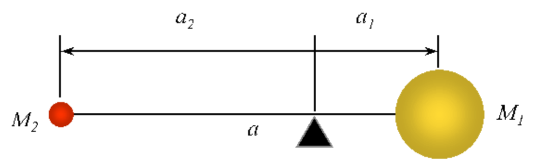
Session 7 Stellar Structure & Energy Sources

Stellar Masses

Recall from Session 4:

$\frac{G(M_1+M_2)}{a^3} = \frac{4\pi^2}{P^2}$; Newton's version of Kepler's 3rd law for circular orbits.

Also note:



Two stars in a binary system will orbit their centre of mass or *barycentre* such that:

$M_1a_1 = M_2a_2 \rightarrow M_1/M_2 = a_2/a_1$

Thus if $M_1 + M_2$ & M_1/M_2 can be found; M_1 & M_2 can be determined – in theory.

The Practice

The Doppler Effect

Stellar spectra – absorption lines

Wavelength shift $\Delta\lambda$:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{v}{c}$$

λ_0 = rest wavelength

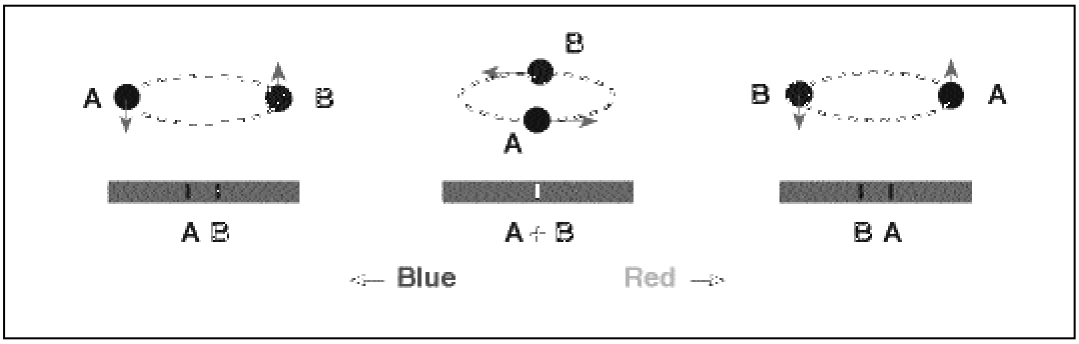
C = velocity of light

v = line of sight or *radial velocity*; +ve for red shift; -ve for blue shift.

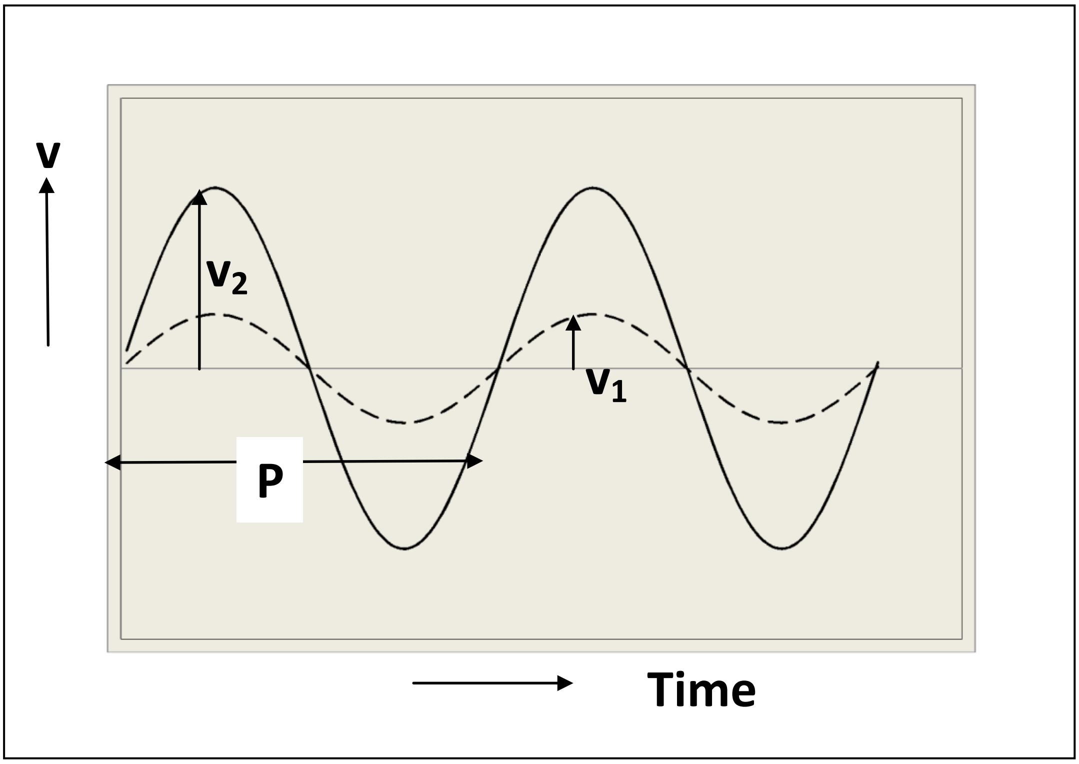
$\Delta\lambda$ determined by imaging star's spectrum alongside 'rest spectrum' traditionally produced by iron arc built into spectroscope (iron arc produces many emission lines).

Double line spectroscopic binaries

Stars too close to be resolved in telescope but spectral lines of both stars are observed.



Doppler shifts → v_1 & v_2; Plot these against time – *radial velocity curve.*



P obtained from radial velocity curve and v_1 & v_1 give a_1 & a_2 (and hence a) from:

$$P = \frac{2\pi a_1}{v_1} = \frac{2\pi a_2}{v_2}.$$

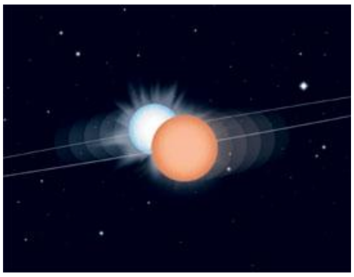
Hence M_1 & M_2 can be determined.

Problem!

We don't know the angle i that the plane of the binary orbit makes with the plane of the sky – result is that $v_{observed} = v_{real} \times \sin i$.

Solution!

Use only double line spectroscopic *eclipsing binaries*. Mutual eclipses of stars cause variation of apparent magnitude over period P $\rightarrow$ *light curve*. Eclipsing binaries ensure i$\simeq$ 90°.



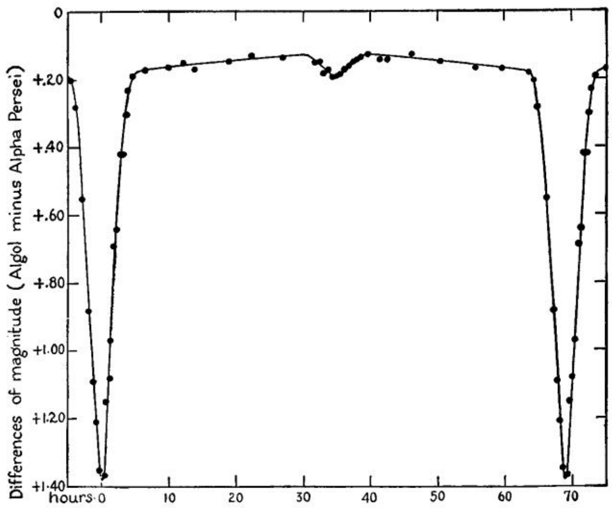


Fig. 2.—Photoelectric light curve of Algol.

Light curve of Algol (β Persei)

Shallow secondary minimum caused by passage of fainter star in front of brighter.

Mass luminosity relation

Most common form – *empirical mass luminosity relation.*

Found by producing calibrated curve of luminosity (if distance known – Session 5) vs. mass for a suitable sample of stars.

It is found that;

$\boldsymbol{Log\left\{\frac{L}{L_{\odot}}\right\} = 3.8 Log\left\{\frac{M}{M_{\odot}}\right\} + 0.08}$; Logs are to base 10.

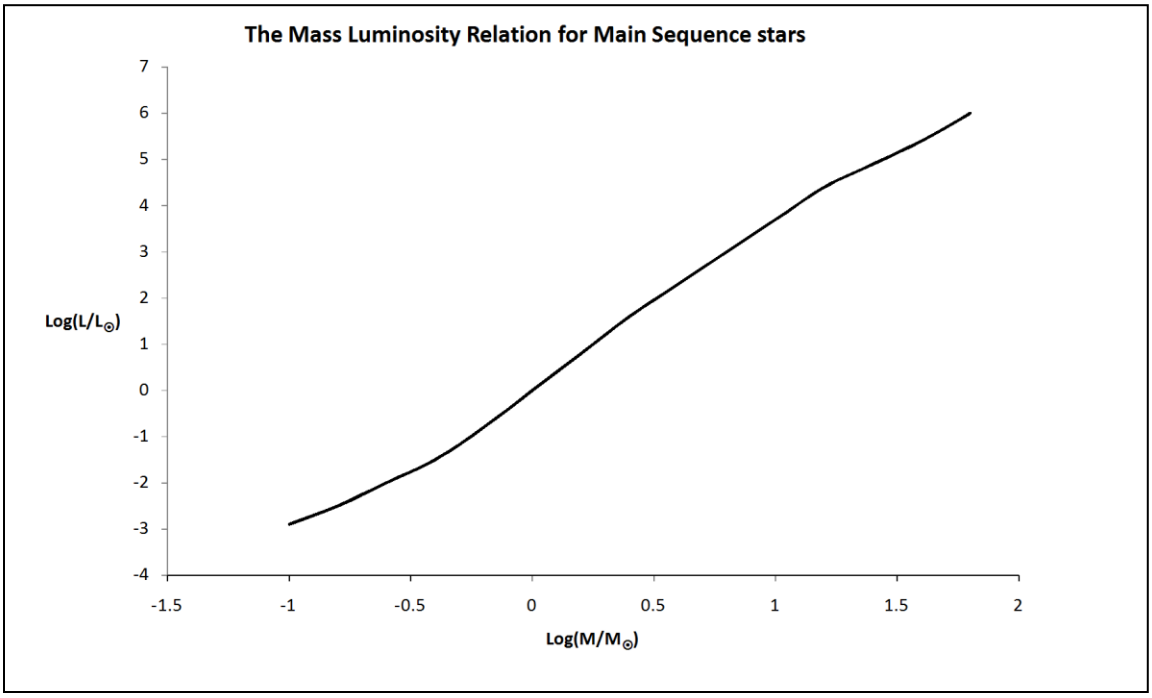
Or:

$$\boldsymbol{Log\left\{\frac{L}{L_{\odot}}\right\} = Log\left\{\frac{M}{M_{\odot}}\right\}^{3.8} + Log\ 1.2}$$

i.e.

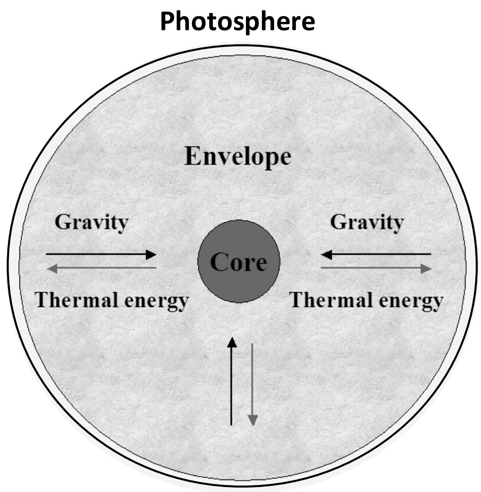
$$\frac{L}{L_{\odot}} = 1.2\left\{\frac{M}{M_{\odot}}\right\}^{3.8}$$

This is for main sequence stars.



Stellar Stability – The Equations of Stellar Structure

Basic anatomy of a star



Stellar core defined as zone where energy is generated.

No energy generation in envelope.

Energy transported through envelope by:

1. Radiative diffusion; i.e. free-free transitions of electrons & atomic nuclei.
2. Convection if temperature gradient is too steep.

Most stars (particularly main sequence) are stable. Stability rests on balance between gravity & thermal energy; i.e. gas pressure (plus radiation pressure for hottest stars) resulting from energy production in core.

Stars above main sequence in HR diagram are usually unstable – next session.

Equations of stellar structure:- Arthur Eddington 1920's

Four main equations plus three supplementary equations.

A Brief Note from Calculus

δx; a small interval of the variable x. dx; an infinitesimal interval of the variable x.

δy; a small interval of the variable y dy; an infinitesimal interval of the variable y.

$\frac{\delta y}{\delta x}$; The rate of change of y w.r.t. x over the small interval δx.

$\frac{dy}{dx}$; The rate of change of y w.r.t. x *at the point x*.

For any function f(x) we know that:

$f(x + \delta x) = f(x) + \frac{df}{dx}\delta x$

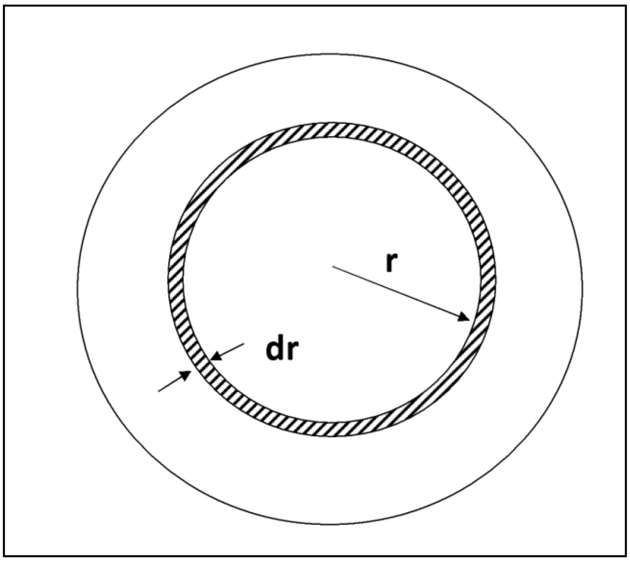
or:

$f(x + \delta x) - f(x) = \delta f = \frac{df}{dx}\delta x$:

which leads to the 'trivial'

(but useful for us here) result:

$df = \frac{df}{dx}dx$



Deal only with spherical star; radius r consisting of series of thin concentric shells of thickness dr. Volume of a shell is then equal to $4\pi r^2 dr$.

- ***Conservation or Continuity of Mass*. This equation describes how the star's mass varies with r.**

  If the total mass enclosed within the star out to radial distance r is M(r) then the total mass of the shell is given by:

  M(r + dr) – M(r) = dM(r) or $\frac{dM(r)}{dr}\boldsymbol{dr}$ = ρ(r)dV = ρ(r) × 4πr^{2}dr

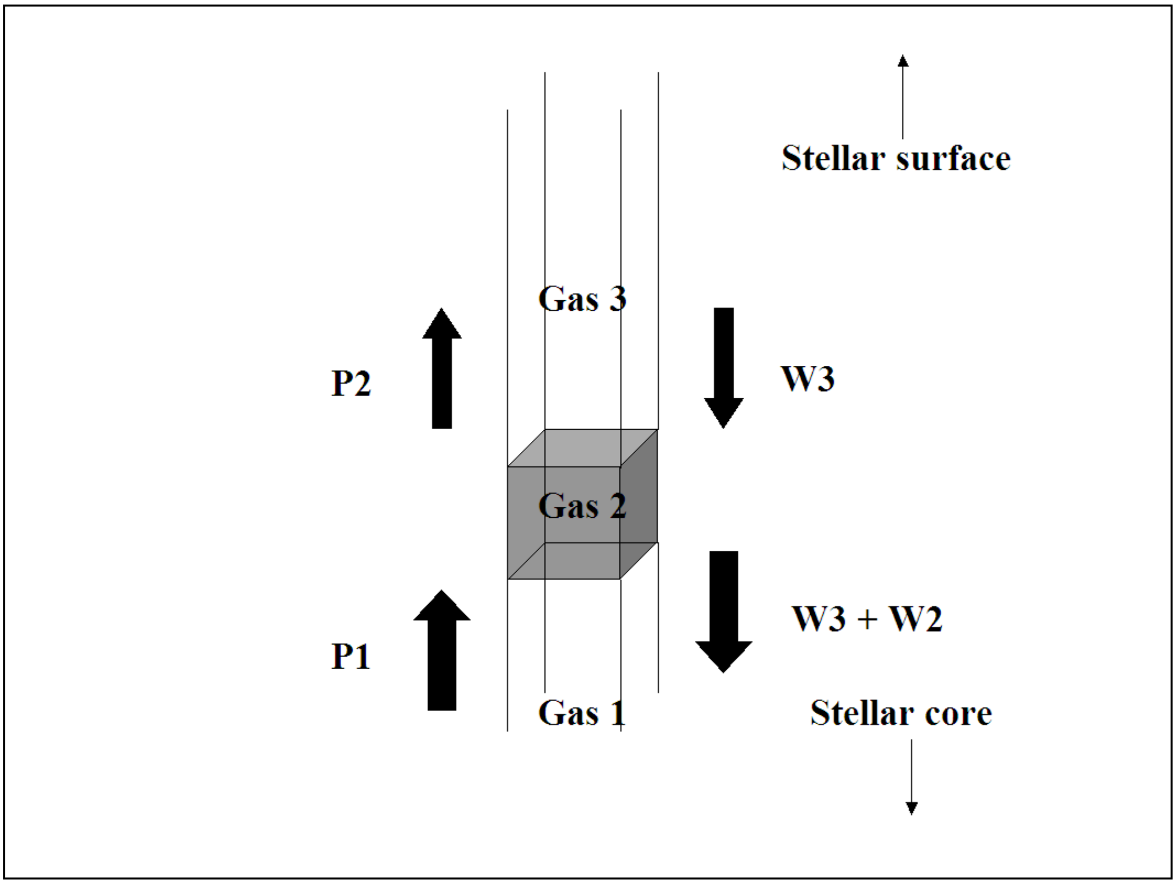
  ρ(r) is the density of stellar material at radial distance r and dV is the volume of the shell.

  i.e. $\frac{dM_r}{dr}$ = ρ(r) × 4πr^2 : *Equation of continuity of mass*

  Here the standard notation is to use M_r to denote the total mass contained within the radial distance r from the centre of the star.

- ***Hydrostatic Equilibrium***

  Illustrated in the figure below. Basically every volume element within the star has to be capable of supporting against gravity, all volume elements directly above it. In turn, these and the volume element itself must be supported by the volume element immediately below. In order for this to be possible, the pressure at the base of the element must be slightly greater than that at the top; in other words there must be a *pressure gradient* within the star with pressure increasing for smaller values of r.



For the gas pressure at the outer and inner surfaces of the shell we have:

P(r + dr) – P(r) = - $\frac{dP}{dr}$dr; where the negative sign indicates that P increases as r decreases. This then equals the (upward) force on a small volume element of the shell of thickness dr and unit cross sectional area. The downward gravitational force on the element is equal; to: $\frac{-GM_r dm}{r^2}$. For hydrostatic equilibrium, the sum of these two forces must equal zero, so:

$\frac{-dP}{dr}\mathbf{dr} + \frac{-GM_r dm}{r^2} = \mathbf{0}$; or:

$\frac{dP}{dr}\mathbf{dr} = \frac{-GM_r dm}{r^2}$.

For the volume element; dm = ρ(r)dr (remember the element has unit cross sectional area) and so:

$$\frac{dP}{dr}\mathbf{dr} = \frac{-GM_r\rho(r)dr}{r^2}$$

Hence

$\frac{dP}{dr} = \frac{-GM_r\rho(r)}{r^2} = -g\rho(r)$: ***Equation of hydrostatic equilibrium***

Where g is the acceleration of gravity at distance r from the star's centre.

- ***Conservation of Energy***

  Stars generate energy but only within their cores. This energy is lost by radiation eventually reaching the star's photosphere. Within the core the luminosity (this is measured in watts as for the star as a whole) will increase as we move through successive shells of increasing 'r' but upon reaching the envelope there is no further energy production. The luminosity is then constant as we move through the envelope to the photosphere.

  So firstly within the core, the luminosity difference between the inner and outer radii of our shell; i.e. the energy generated within it will be:

  $L(r + dr) - L(r) = \frac{dL(r)}{dr}dr$;

  Calling the energy generated per kg of stellar material 'ε', the total energy generated within the shell is also equal to:

  $\varepsilon\rho(r)4\pi r^2 dr$;

  And so:

  $\frac{dL_r}{dr} = \varepsilon\rho(r)4\pi r^2$:

  Here we adopt the similar convention that L_r is the luminosity emerging at radius r.

- ***Energy Transport***

  The energy generated in a star's core must be able to flow through the envelope to the photosphere. It does this by radiation; stellar material consists of plasma and in the deeper hotter regions of a star, atoms exist in a highly ionised state so that free electrons greatly outnumber atomic nuclei. Energy generated in the core is initially in the form of gamma ray photons; these are constantly absorbed and re-emitted (usually as a series of lower energy photons) in free-free transitions.

  The result is that the rate of flow of energy through unit area of our thin shell (the flux F(r)) is proportional to the temperature difference across the shell's thickness. The proportionality constant which we can call λ (nothing whatsoever to do with wavelength) depends on the temperature itself, the density and the chemical composition of the stellar material.

  We have:

  $$F(r) = -\lambda \frac{dT(r)}{dr};$$

  Once again the negative sign indicates that the temperature decreases with increasing r. So:

  $$L_r = F(r) \times 4\pi r^2 = -\lambda \times 4\pi r^2 \frac{dT(r)}{dr}$$

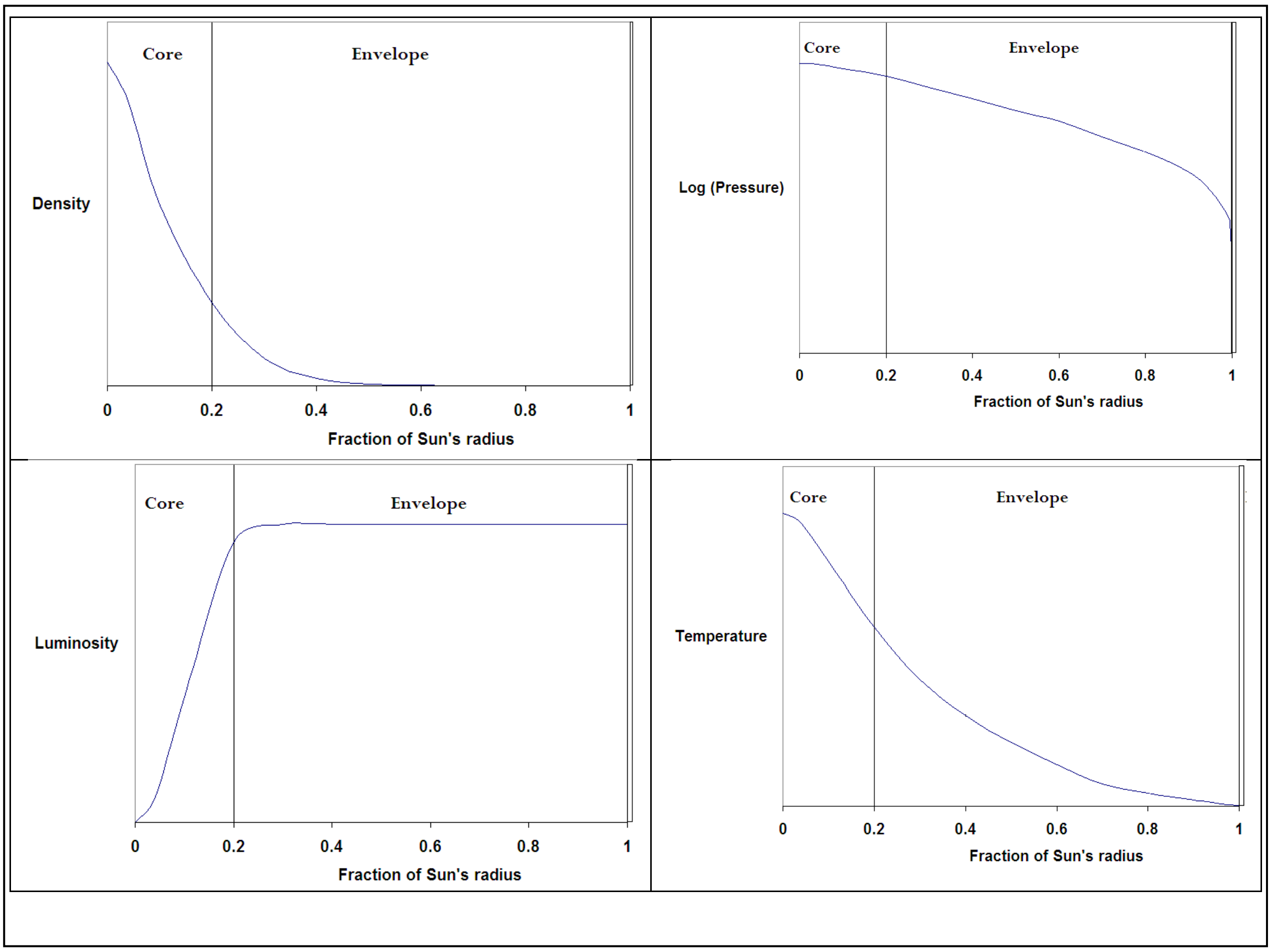
Hence: $$\frac{dT}{dr} = -\frac{L_r}{4\pi r^2 \lambda}$$

The first of the three supplementary equations is usually referred to as the 'equation of state' and it gives the pressure within the stellar material in terms of density, temperature and chemical composition.

The second equation specifies the opacity of the material within the star. For theoretical astronomers seeking to 'construct' model stars, this is horrendously complex because the opacity which depends on temperature, density and chemical composition must be calculated for all wavelengths.

Finally the third equation simply states how much energy is extractable from each kilogram of stellar core material.

The plots below show how the various fundamental stellar parameters vary with r for the Sun.



The final thing to say here is that stable stars are for the most part stars which lie along the Main Sequence in the HR Diagram. Stars which lie above the Main Sequence are very often unstable for some reason.

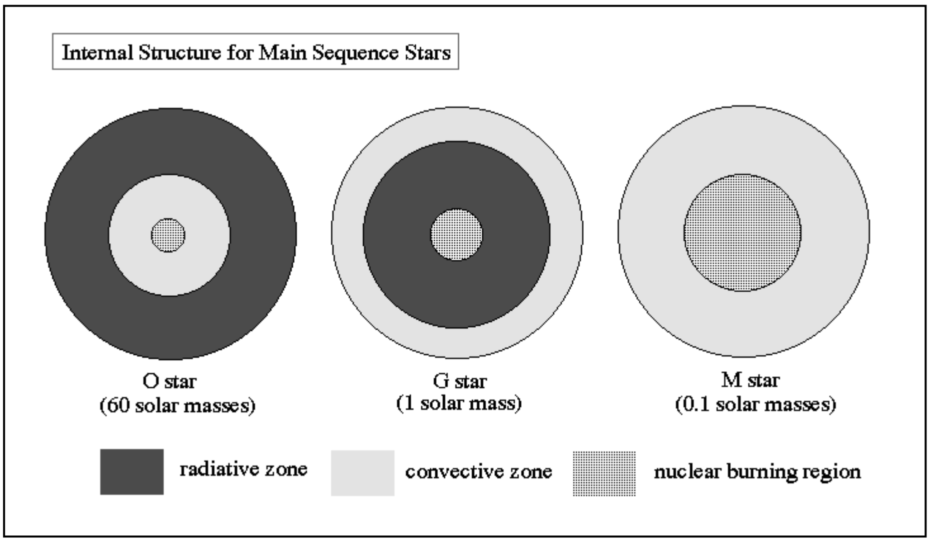
Energy transport in stellar envelope

- Radiative diffusion; free-free transitions of atomic nuclei &especially electrons. Temperature gradient not too steep; stellar material remains more or less in same location.
- Convection; bulk motion of stellar material to higher cooler layers. Dominates when temperature gradient too steep.

Hot stars; e.g. Class O – convective near core then radiative I outer layers as temperature gradient becomes less steep.

Solar type tars; e.g. Class G – radiative near (cooler than Class O) core; becomes convective in outer layers as opacity (Session 9) of stellar material increases → steep temperature gradient.

Cool stars; e.g. Class M – convective throughout envelope; high opacity → steep temperature gradient.



Stellar energy sources

Sun's luminosity = 3.826×10^{26}watts. Energy must be generated at this rate to keep the sun shining in all for ~ 10^{10} years.

- Fossil fuel; ~ few × 10^4 years.
- Continued gravitational contraction ~ 2×10^7 years
- Fusion of hydrogen to helium ~ 10^{10} years.

Nuclear Reactions in the Stellar Core

Hydrogen converted to helium in core of main sequence stars.

Four hydrogen atoms each of mass 1.674×10^{-27}kg and hence a combined mass of 6.696×10^{-27}kg are converted into one atom of helium with a mass of 6.645×10^{-27}kg. The mass deficit of 5.1×10^{-29}kg (0.0076 of mass of 4 hydrogen atoms) is converted into $5.1 \times 10^{-29} \times 9 \times 10^{16} = 4.59 \times 10^{-12}$joules of energy ($E = mc^2$).

Assume 10% of sun's hydrogen available for fusion.

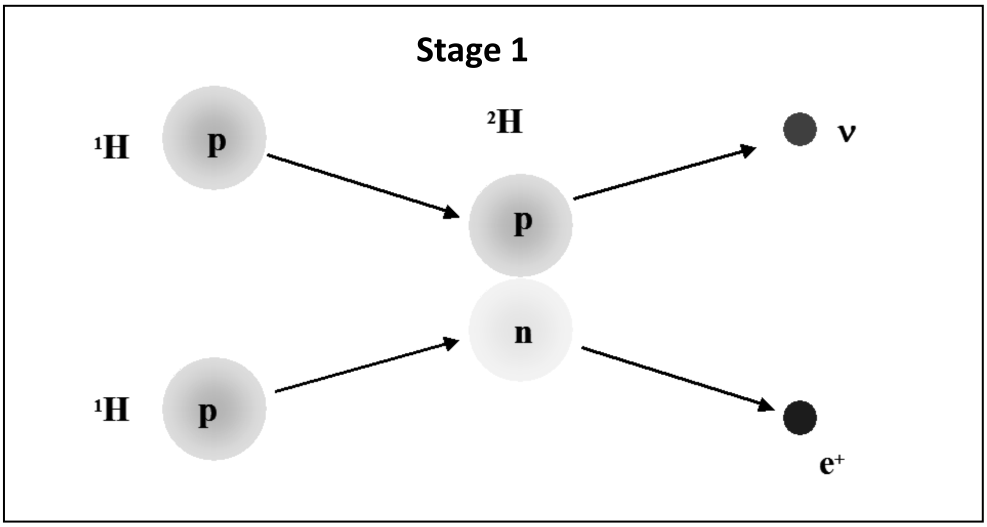
$E = 0.0076 \times 0.1 \times 2 \times 10^{30} \times 9 \times 10^{16} = 1.37 \times 10^{44}$ joules.

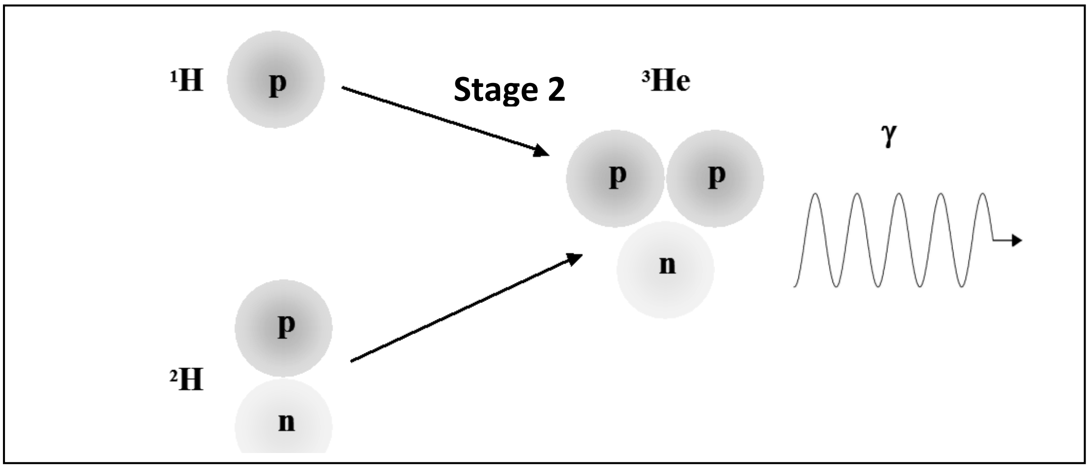
Sun's lifetime = $E/L_{\odot} = \frac{1.37\times10^{44}}{3.826\times10^{26}} = 3.58 \times 10^{17} sec$

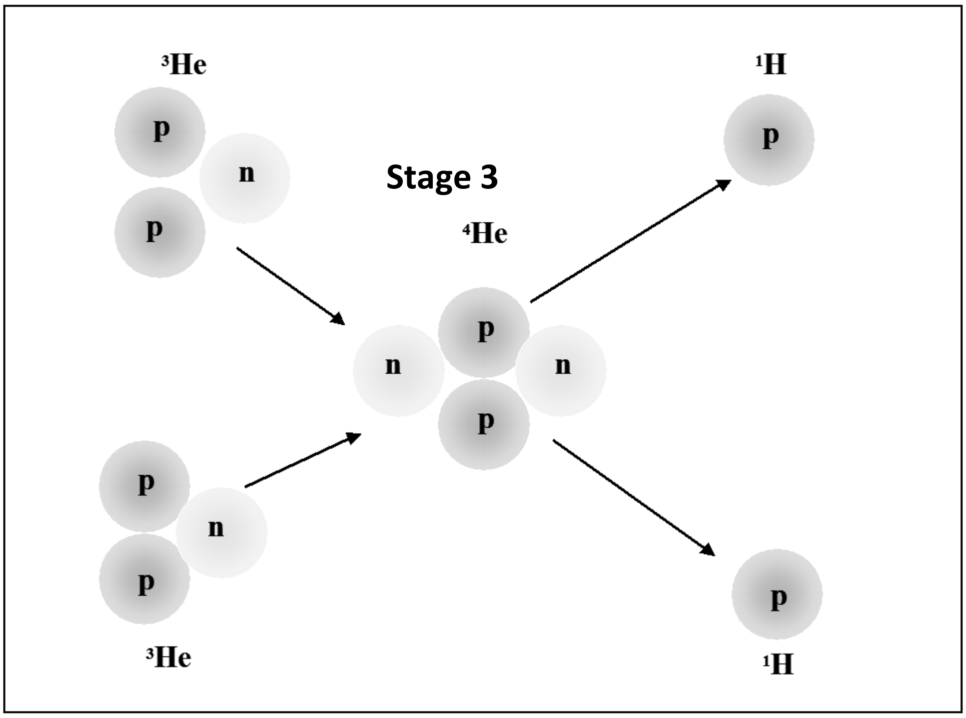
i.e. 1.13×10^{10} years.

The proton-proton (p-p) chain

For all but the hottest stars the dominant process is called the *proton-proton or p-p chain*. This takes place in three stages.







In symbols:

$$^{1}H + {}^{1}H \rightarrow {}^{2}H + \nu + e^{+}$$

$$^{1}H + {}^{2}H \rightarrow {}^{3}He + \gamma$$

$$^{3}He + {}^{3}He \rightarrow {}^{4}He + {}^{1}H + {}^{1}H$$

(Learn this & remember it).

Notice that only in the second stage is energy released and that two protons emerge after the third stage to form the next 'link' in the chain.

For the hottest stars a more complex series of reactions takes place. This is called the carbon nitrogen oxygen or *CNO cycle.* The process still turns hydrogen into helium but here carbon nitrogen & oxygen nuclei act as catalysts.

$$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma$$

$$^{13}N \rightarrow {}^{13}C + e^{+} + \nu$$

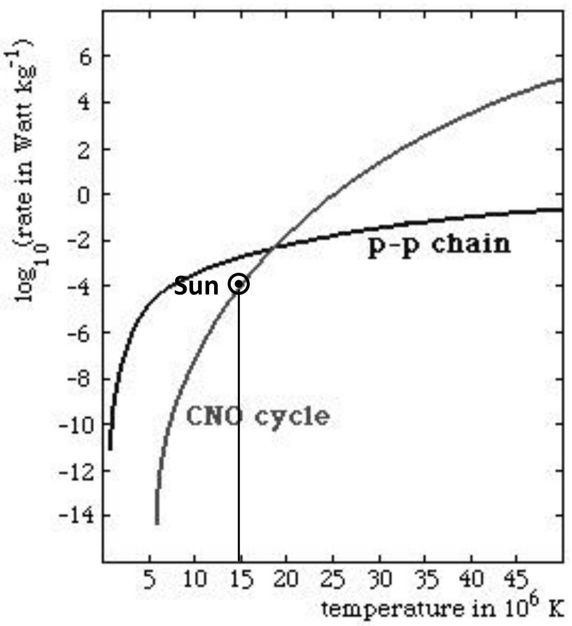
$$^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma$$

$$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$$

$$^{15}O \rightarrow {}^{15}N + e^{+} + \nu$$

$$^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He$$

(No need to learn this).



Temperature dependence of p-p chain & CNO cycle