ASTROPHYSICS Yr 2 2011-2012

Session 9 Radiative transfer

Radiation from a star suffers absorption, re-emission & scattering before reaching the telescope. These processes take place in:

- The stellar envelope & surface layers.
- The interstellar medium; including nebulae.
- The Earth's atmosphere.

Reminder: Flux = rate at which energy is emitted from (e.g. photosphere of star) or falls on (e.g. detector) 1 sq. metre – *watts per sq. metre.*

More rigorously: Flux is actually *net rate of flow of energy through 1m^2*; e.g. 1 sq. metre in space will experience a flow of energy from all directions → in an *isotropic radiation field*; net flow; i.e. flux = 0.

Specific intensity 'I'; is essentially flux confined to a specific direction & is generally used in radiative transfer calculations. However for stars it is flux that is measured; so can simply use f.

Overall effect of absorption & re-emission processes; photons become degraded; i.e. one higher energy photon → several lower energy photons.

Absorption of Starlight by the Earth's atmosphere

The flux $\mathbf{f_0}$ which reaches the top of the atmosphere is reduced to $\mathbf{f_1}$ at ground level in such a way that:

$$f_1 = f_0 e^{-\tau}$$

Remember this equation.

The quantity 'τ' is called the *optical depth* along the light's path through the atmosphere; generally the shorter the light path (i.e. the smaller the zenith distance of the star) the smaller the optical depth which results in less reduction in the flux.

If we call the apparent magnitudes of the star at the top and bottom of the atmosphere; m_0 & m_1 respectively, then:

$$2.512^{(m_1 - m_0)} = \frac{f_0}{f_1} = e^{\tau}$$

Satisfy yourself that this term results from re-arranging the above equation.

This time we take logs to base 'e' (Ln on your calculator);

$(m_1 - m_0) \times 0.921 = \tau$

So:

The change in magnitude (m_1- m_0) = 1.086τ.

The optical depth has wide application in astrophysics; an absorbing medium with an optical depth of 1 or more is called '*optically thick',* otherwise it is *optically thin.*

Linear absorption coefficient; κ (opacity)

Drop in flux df as a result of absorption along (small) unit path length given by:

$$\delta f = -\kappa f_0$$

-ve sign → δf is a decrease in initial flux value $f._0$

Linear absorption coefficient κ depends on chemical makeup of absorbing medium together with density & temperature. It is also wavelength dependent; i.e. should write κ_λ but just use κ here while acknowledging this.

Along a path length ℓ;

δf; = -$\kappa\ell f_0$ but only if κ is constant along ℓ – this is not usually the case.

The quantity κℓ is the *optical depth* τ.

More rigorously with calculus:

df = -κfdℓ; dℓ is a tiny length interval along the light path.

So:

$$\frac{df}{f} = -\kappa dl$$

Over the total path length ℓ:

$$\int_{f_0}^{f} \frac{df}{f} = -\int_0^l \kappa dl$$

Ln f – Ln f_0 = $Ln\frac{f}{f_0}$ = -κℓ = τ In calculus; $\int \frac{1}{x}dx$ = Ln x

$f = f_0 e^{-\kappa\ell} = f_0 e^{-\tau}$

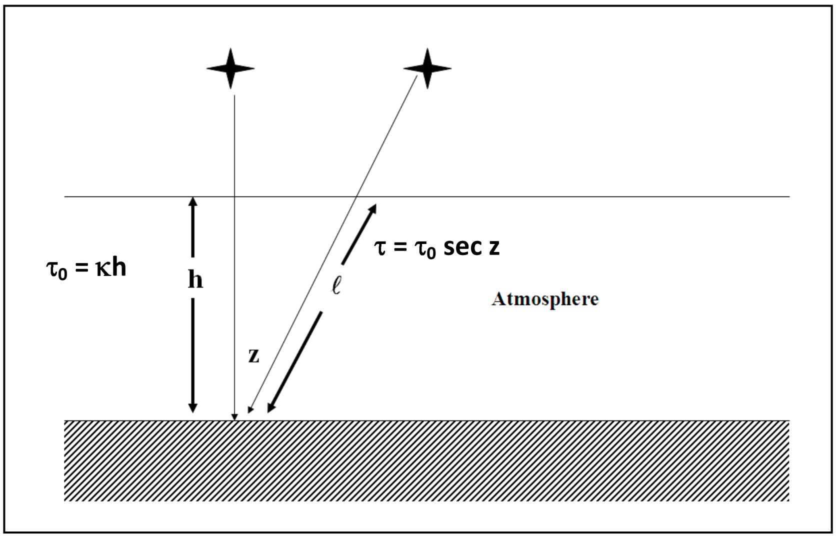
Generally κ will be some function (not necessarily a straightforward one) of ℓ and calculating τ is often difficult.

Determination of 'top of atmosphere' magnitude; m_0 – Bougeur's method

Make a series of observations of same star at varying zenith distance **z**.

Provided $z < 60^o$, can ignore curvature of surface of Earth & top of atmosphere (*plane parallel approximation*).

Vertical height of atmosphere = h & (as will be seen) can ignore varying density & in turn opacity of atmosphere with height; κ then is essentially the mean opacity.



At zenith optical depth $\tau_0 = \kappa h$.

At zenith distance z; optical depth = $\kappa\ell = \kappa h/\cos z = \tau_0/\cos z = \tau_0 \sec z$.

Output of CCD camera $\propto$ flux f from star so simply call it f.

Flux at top of atmosphere = f_0 so flux received;

$$f = f_0 e^{-\tau}$$

$$\frac{f}{f_0} = e^{-\tau}$$

Take logs to base 10:

$\text{Log } f - \text{Log } f_0 = -\tau \text{Log } e$; Log e = Log 2.718 = 0.4343.

$\tau = \tau_0 \sec z$

$\text{Log } f - \text{Log } f_0 = -0.4343\tau_0 \sec z$

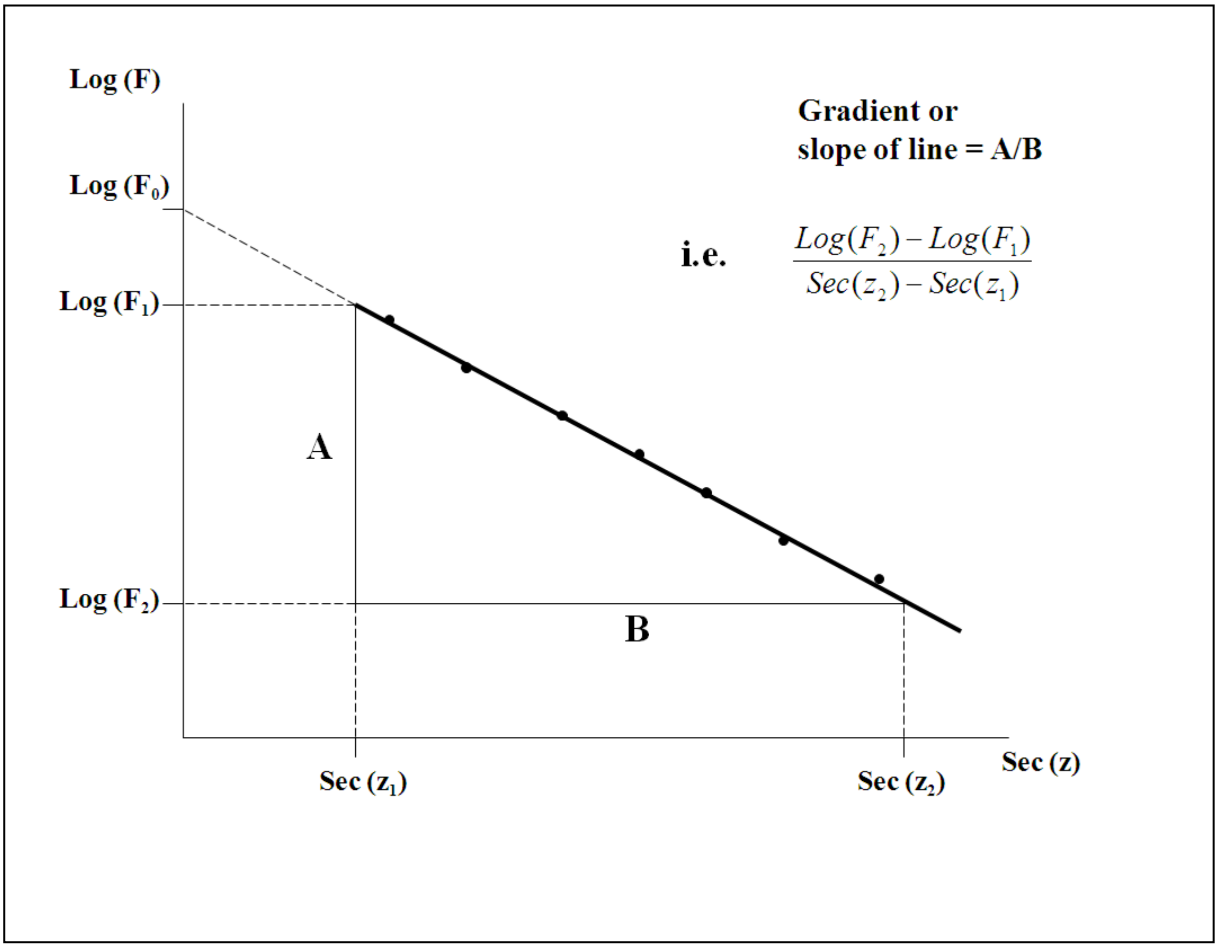
$\text{Log } f = -0.4343\tau_0 \sec z + \text{Log } f_0$; i.e. *equation of a straight line.*

Plot Log f vs. sec z.

$$\frac{-\,slope\ of\ plot}{0.4343} = \tau_0$$; i.e. vertical optical depth of atmosphere.

Intercept on 'y' axis → flux at top of atmosphere.

Note; actual observation of star at zenith not needed (most stars don't cross zenith anyway).

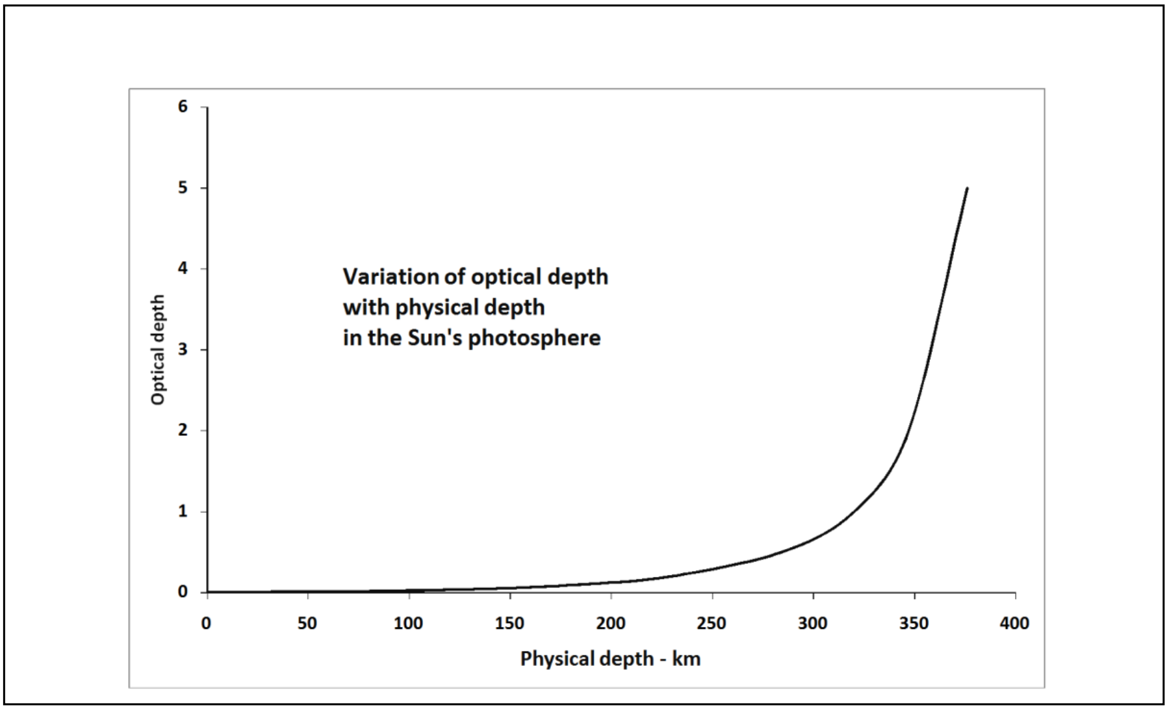


Carry out similar set of observations for Vega or suitable 'standard star' of known (top of atmosphere) magnitude. (Note: Vega is *defined to have* a top of atmosphere magnitude of 0.0).

Two derived f_0 values → top of atmosphere magnitude for star.

Absorption in the Sun's photosphere

Situation similar to that in Earth's atmosphere but in reversed form; i.e. optical depth determines how deep we see into sun's surface layers.



The radius of the Sun is 6.96×10^5 km, yet at a depth of only a little over 300 km, the photosphere has become optically thick. This means that radiation which originates at this depth has diminished by a factor of e^{-1} or 0.368 by the time it reaches the surface and that which comes from a depth of only around 350km has dropped to e^{-5}; i.e. 0.007 of its initial value. Radiation from deeper in the photosphere gets reabsorbed/scattered before it can reach the surface or put another way a photon's mean free path is much less than the distance to the surface. Only when this distance becomes comparable to the mean free path does the photon have a good chance of escaping; so all that we see of a star like the sun comes from the very topmost layer.

Limb Darkening

Both the temperature and density of the Sun's photosphere vary; the temperature ranging from around 9,000K at the bottom layer to around 4,500K at the top. However because the thickness of the Sun's photosphere is so small compared to the Sun's radius, we can assume an average temperature and density throughout the photospheric layer. The optical depth is then only a function of the light path distance ℓ ; so $\tau = \kappa\ell$. Once again astronomers make use of the so called '*plane parallel' approximation* (i.e. the uppermost and lowermost photospheric layers are regarded as parallel layers) in order to simplify any analysis of what's going on.

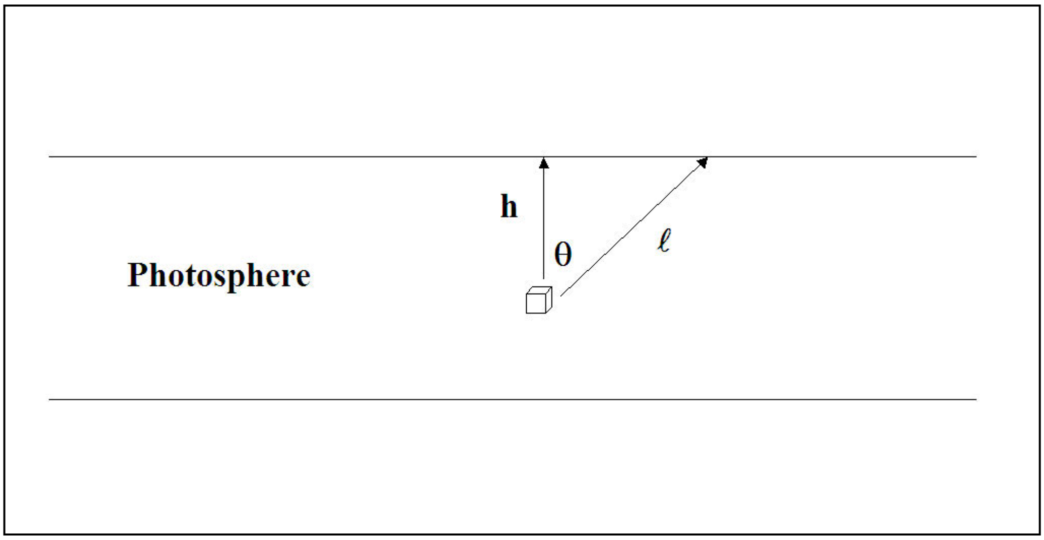
The figure below shows a small volume element situated at vertical depth **h** below the top of the photosphere. This element will emit radiation in all directions (including many which take the radiation straight back down into the Sun – for 'recycling') one of which is indicated. The optical depth τ_ℓ is given by:

$$\tau_\ell = \kappa \frac{h}{\cos\theta}$$

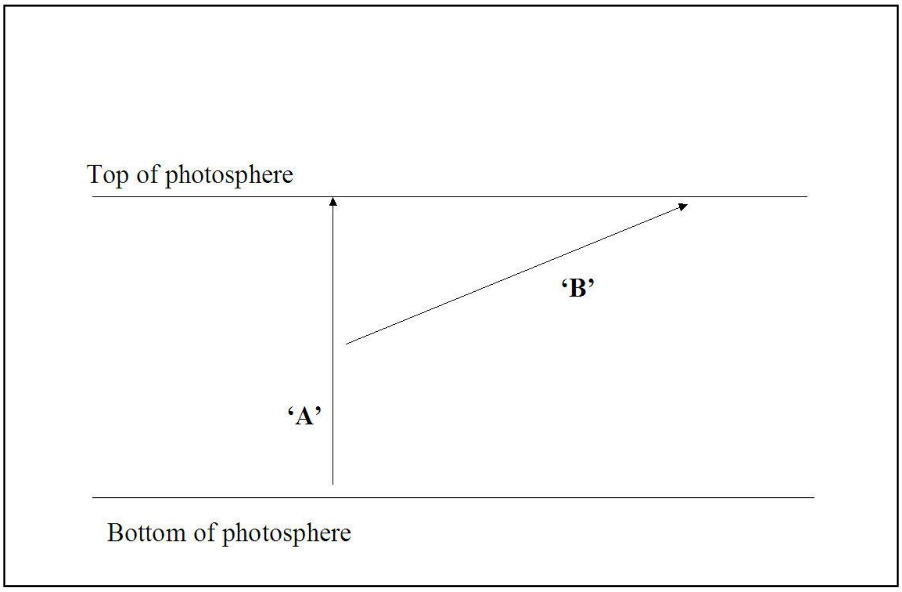
But the optical depth τ_0 straight down through the photosphere is given simply by κ**h**.

So:

$$\tau_\ell = \frac{\tau_0}{\cos\theta}$$

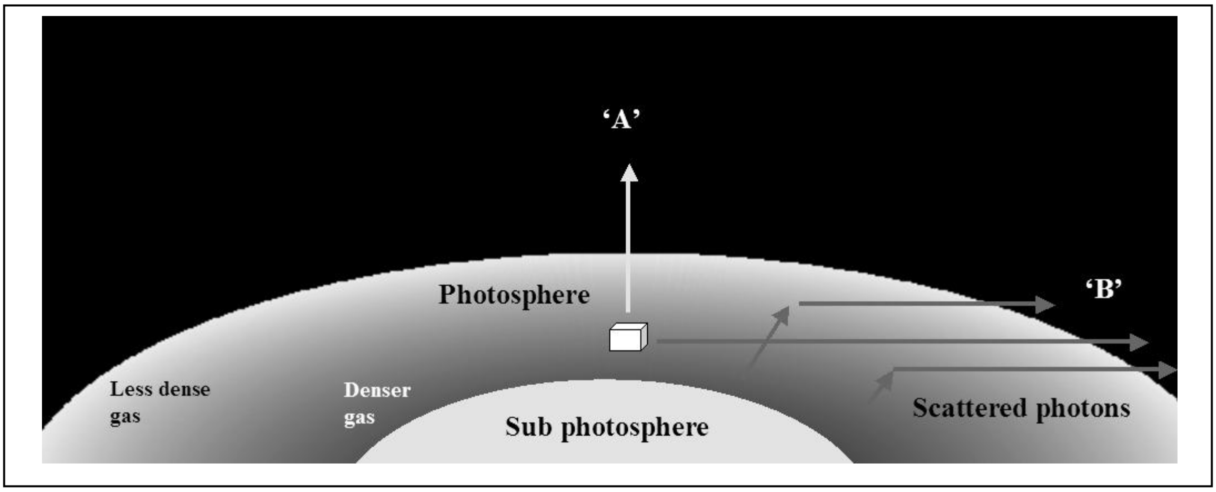


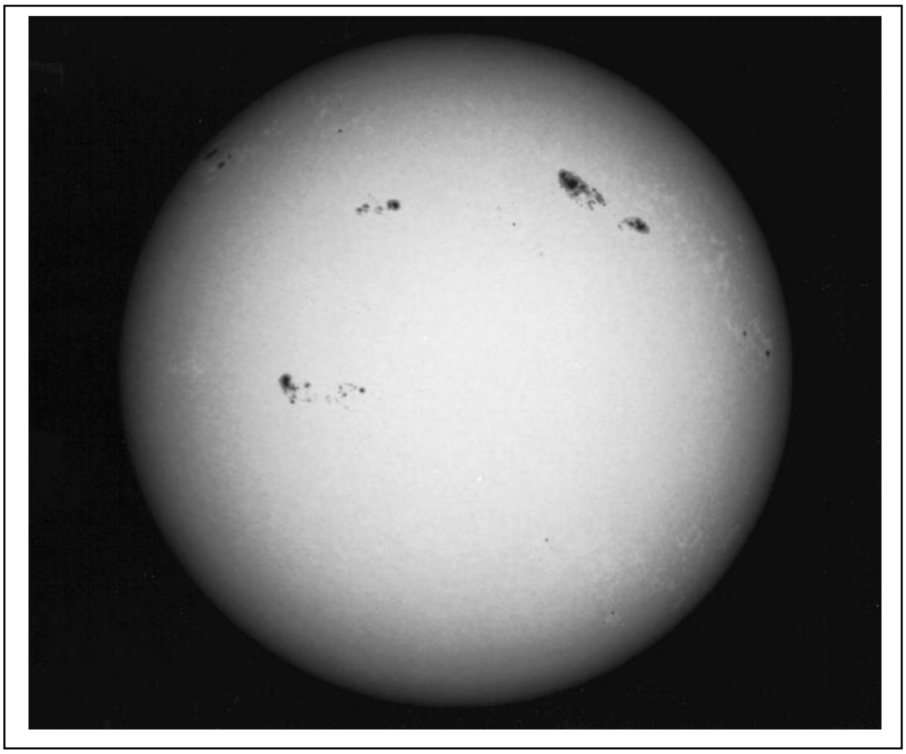
The smallest optical depth along 'h' enables us to see to a greater physical depth; i.e. down to regions of the Sun which are hotter and brighter. In the figure below, the two path lengths 'A' and 'B' have the same optical depth.



Below we see (not to scale and with the depth of the photosphere highly exaggerated) how looking along path 'A' we are observing the central region of the Sun's visible disc, whereas along paths like

‘B’ we are looking towards the Sun’s outer edge or ‘limb’. The result is that the Sun’s limb region appears darker than the central region – an effect not surprisingly called *limb darkening*. Indirect evidence suggests that some other stars also exhibit limb darkening.

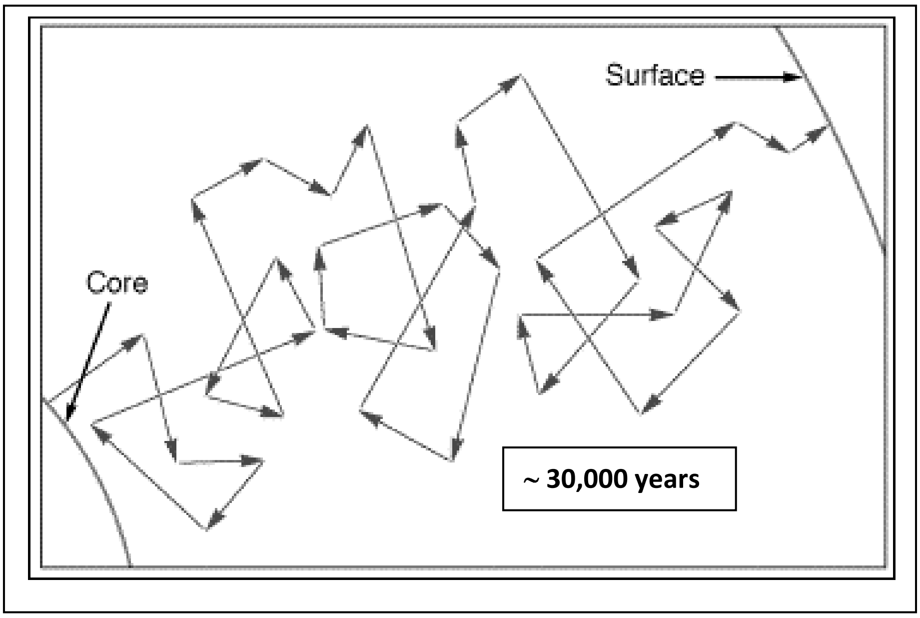




A final important point here is that the above image could give the impression that in the darker limb region we actually ‘see’ the physical extent of the photosphere itself. This is not so; on the scale of this image the physical depth of the photosphere would be vastly less than the darkened area.

Radiative transfer in a stellar envelope (very briefly)

Radiation starts as γ ray photons in stellar core. These 'random walk' their way to star's photosphere. Become degraded; i.e. converted to large numbers of lower energy photons.



Radiative transfer **within a nebula**

A nebula is very far away; can only observe that beam of photons which is directed towards Earth.

Photons originate in hot stars; e.g. class O or B stars in a galactic nebula or central star of a planetary nebula. Many photons initially of high energy; i.e. UV.

Deal with only one single wavelength λ (strictly; a tiny wavelength interval $d\lambda$ centred on λ itself).

Observed radiation from nebula consists of:

- Photons from stars within or behind the nebula which survive to escape the nebula.
- Emission from the nebula material itself which results from recombination/de-excitation of atoms following ionisation or excitation by high energy photons.
- Photons scattered into the direction towards Earth by all parts of the nebula.

Nebular & scattered emission will themselves undergo further absorption & scattering within the nebula.

Scattering

By gas atoms & dust particles; photon 'survives' – may suffer Doppler wavelength shift due to motion of atom. Scattering changes direction of photon.

Atom provides 'scattering target' for photon; probability of scattering $\propto 1/\lambda^4$; i.e. blue light scattered much more than red light.

This causes e.g. Earth's blue sky due to scattered sunlight. Also gives rise to *reflection nebulae* (better name would be scattering nebulae) such as NGC1977 in Orion – notably blue in colour.

Can define *linear scattering coefficient* 'σ' in same way as linear absorption coefficient. This is simply added to absorption coefficient κ to give the *extinction coefficient* 'χ' i.e. *opacity* of medium.

$\chi_\lambda = \kappa_\lambda + \sigma_\lambda$; Note wavelength dependence.

Observed flux (beam of photons) will contain photons from all parts of the nebula which have been scattered into direction towards Earth.

Reflection nebula NGC1977 in Orion. In these nebulae, stars are not hot enough to produce large numbers of high energy UV photons which are needed to ionise the surrounding gas; so photons are scattered instead.

Absorption

Photon is destroyed:

- Ionises or excites target atom.
- Warms up dust grain.

Warmed up dust grains re-emit in infra red so original photon broken down into lower energy IR photons. Scattering coefficient low for these photons → optical depth to boundary of nebula low → photons escape to be observed. Infra red astronomers can 'see' to greater depths in galactic dust clouds.

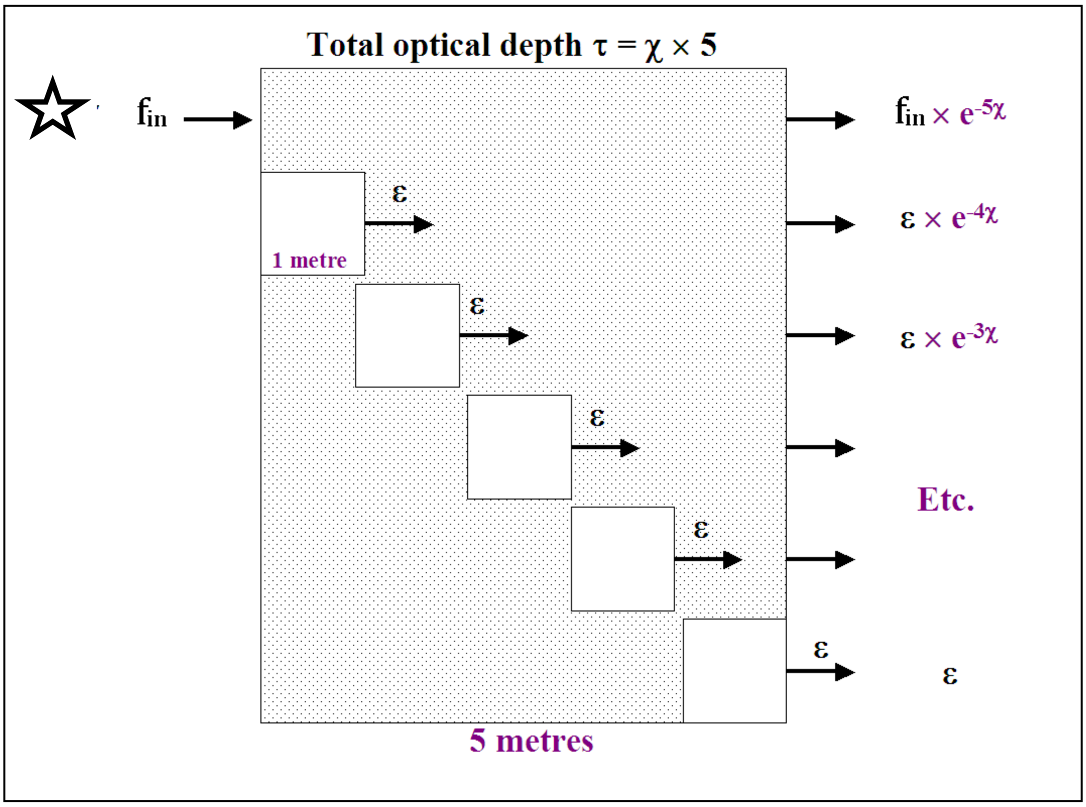
If cloud of dust is present around a star → part of star's optical spectrum diminished & part of infra red spectrum enhanced – *infra red excess.*

Classic case is Vega.

Ionisation/excitation → photon re-emitted but as several lower energy photons. These in turn can be absorbed & re-emitted as photons of lower energy still. This means that nebula material itself can contribute to emission at e.g. optical wavelengths by 'recycling' UV photons from hot star.

Absorption & emission by nebula material

Best to consider (very) simple example



Take a slab shaped nebula:

- Uniform density throughout & has sharp boundary.
- Uniform extinction coefficient χ throughout for observed wavelength λ
- Star on far side of nebula; incident flux = f_{in}.

Total optical depth τ through nebula = 5χ.

Take a series of 1m^3 cubes as shown; Assume ε watts emitted by each cube in direction toward Earth. This is combination of emission & scattering as outlined above. 'ε' *is called the emission coefficient.*

Emission from each cube suffers absorption as shown above.

Total radiation observed at λ = sum of emergent radiation from all volume elements plus that from star.

Finally; including the above processes, it can be shown that:

$f_{out} = f_{in}e^{-\tau} + \frac{\varepsilon}{\chi}(1 - e^{-\tau})$ - Note: not necessary to prove this here.

τ is the optical depth through the entire nebula.

The quantity ε/χ is called the *source function.*

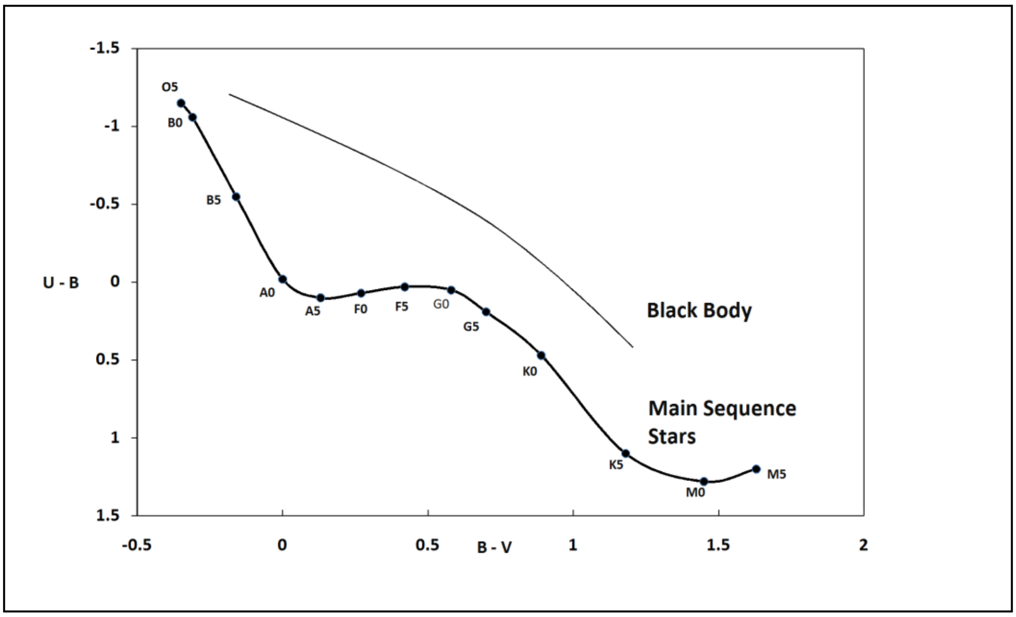
Absorption etc. in the ISM

Investigations by Trumpler (1920's) showed that interstellar space contains material that dims starlight – *interstellar extinction*. Both absorption & scattering predominantly affect short wavelength photons → starlight becomes reddened with increasing distance – *interstellar reddening.*

This affects U – B & B – V colour indices for stars (Session 5).

The Colour-Colour Diagram

Observational H-R diagram plots M_V vs. B – V colour index. However by plotting the U – B colour index against B – V (this is easy to do because these are directly observable quantities) we produce the *Colour-Colour diagram* which looks like this.



Colour Excess

Interstellar extinction will affect the B – V and U – B colour indices for stars. Call the unreddened magnitudes for a star; U_0, B_0 and V_0 ; interstellar extinction results in all these magnitudes being fainter; i.e. they will all have larger numerical values. Furthermore the U magnitude will be affected more than the B magnitude which in turn will be affected more than the V magnitude.

Hence:

$(U - U_0) - (B - B_0) > 0$

And

$(B - B_0) - (V - V_0) > 0$

We can rearrange both of these:

$(U - B) - (U_0 - B_0) > 0$

And:

$(B - V) - (B_0 - V_0) > 0$

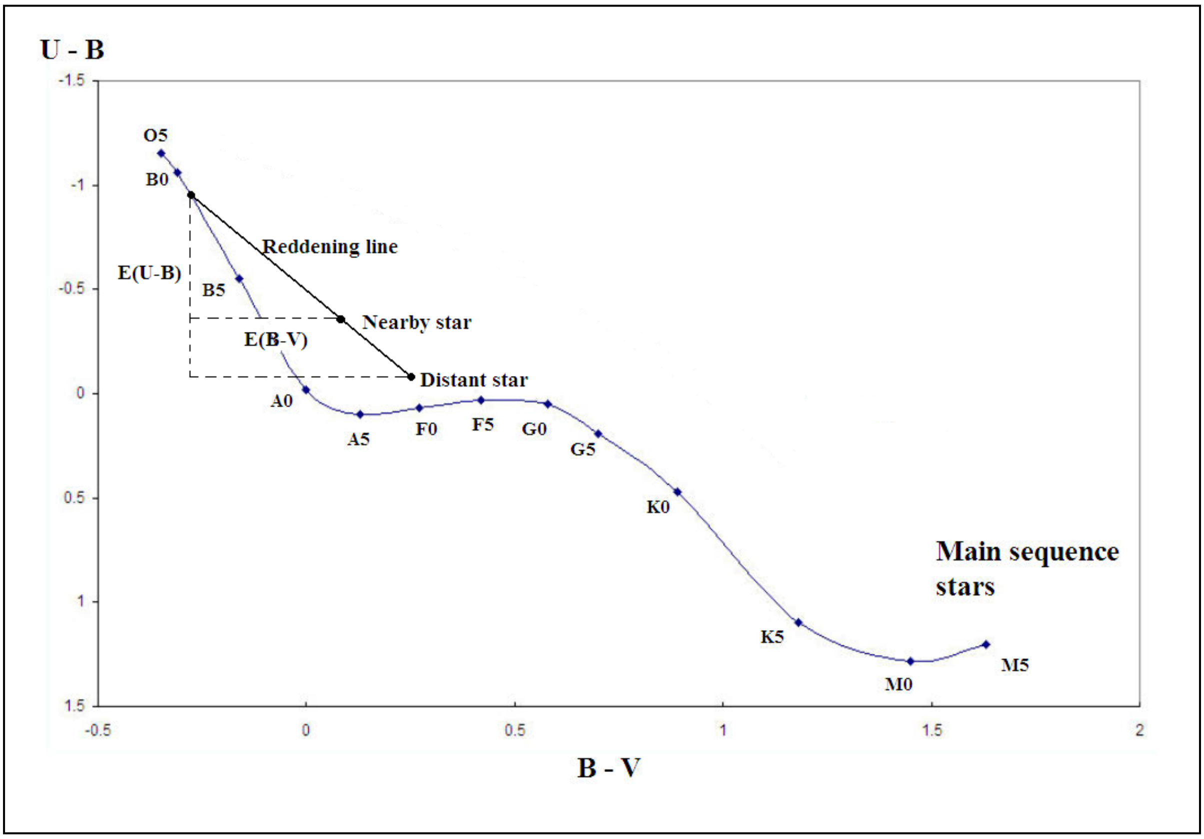
These equations give us the difference between the colour index, that a star is observed to have and that which it would have, if there were no interstellar extinction. Difference in the colour index is called the *colour excess* for the star; it is always a positive number, which gets bigger with increased interstellar extinction and it is written as E(B – V) etc. So:

$E(U - B) = (U - B) - (U_0 - B_0)$

$E(B - V) = (B - V) - (B_0 - V_0)$

The effect of interstellar reddening is to shift a star's position on the colour-colour diagram, down and to the right. What's more it has been found that for early type (i.e. spectral class O, B & A) stars:

$\frac{\boldsymbol{E(U-B)}}{\boldsymbol{E(B-V)}} = \mathbf{0.72} + \mathbf{0.05}(\boldsymbol{B-V})$; or just 0.72 to a first approximation. Two stars of the same (unreddened) spectral type which due to differing distances suffer different degrees of interstellar reddening will lie at two points on the same line on the colour-colour diagram. This line in turn intersects the colour- colour plot itself at the stars' unreddened value and is called the *reddening line.*



The Reddening Free Parameter 'Q'

For a star whose observed magnitudes are; U, B & V,

Define 'Q' as:

$Q = (U - B) - 0.72(B - V)$

$E(U - B) = (U - B) - (U_0 - B_0)$

$E(B - V) = (B - V) - (B_0 - V_0)$

So:

$(U - B) = E(U - B) + (U_0 - B_0)$

And:

$(B - V) = E(B - V) + (B_0 - V_0)$

Then $Q = (U_0 - B_0) - 0.72(B_0 - V_0) + E(U - B) - 0.72E(B - V)$

But:

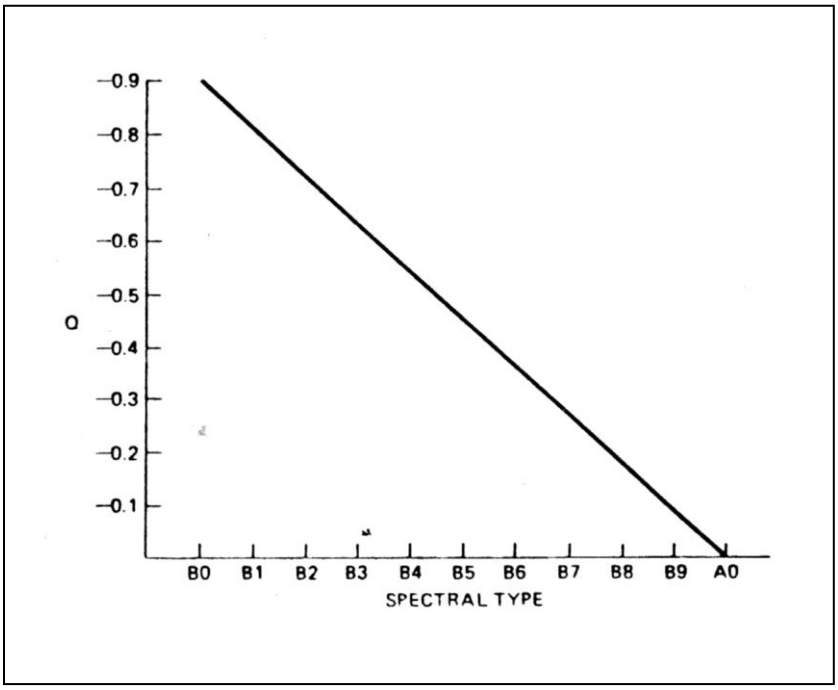
$$\frac{\boldsymbol{E(U-B)}}{\boldsymbol{E(B-V)}} \cong \mathbf{0.72}$$

Hence E(U – B) – 0.72E(B – V) = 0

And:

$Q = (U_0 - B_0) - 0.72(B_0 - V_0)$ *which is independent of interstellar reddening*

The value of Q is plotted below for stars of spectral type B0 to A0.



The value of Q can be used to determine the intrinsic; i.e. unreddened spectral class of early type stars.