UNIVERSITY OF SALFORD **CRN: 13670**

School of Computing, Science & Engineering

SEMESTER ONE EXAMINATION

Programme(s):

MPhys Physics
MPhys Pure & Applied Physics
MPhys Physics with Studies in North America
BSc (Hons) Physics
BSc (Hons) Pure & Applied Physics

Block Code(s)

MP/P/F2 MP/PAP/F2 MP/PN/F2
S/PAP/F2 S/P/F2

ASTROPHYSICS

Instructions to Candidates

Time allowed: 1 hour 30 minutes

Answer the question in **Section A** **AND** any **TWO** questions from **SECTION B**

The question in Section A carries 40 marks
Each question in Section B carries 30 marks

You are advised to spend about **40 minutes on Section A**
and **50 minutes on Section B**

Standard List of Physical Constants provided

STANDARD LIST OF PHYSICAL CONSTANTS

Acceleration due to gravity,	g	=	9.81 m s^{-2}
Astronomical Unit,	1 AU	=	1.496×10^{11} m
Atomic mass unit,	u	=	1.66×10^{-27} kg
Avogadro constant,	N_A	=	$6.02 \times 10^{23} \text{ mol}^{-1}$
Bohr magneton,	μ_B	=	$9.27 \times 10^{-24} \text{ J T}^{-1}$
Boltzmann constant,	k_B	=	$1.38 \times 10^{-23} \text{ J K}^{-1}$
Electron charge,	e	=	1.60×10^{-19} C
Electron rest mass,	m_e	=	9.11×10^{-31} kg
Faraday constant,	F	=	$9.65 \times 10^{4} \text{ C mol}^{-1}$
Gas constant,	R	=	$8.3 \text{ J mol}^{-1} \text{ K}^{-1}$
Gravitational constant,	G	=	$6.672 \times 10^{-11} \text{ N m}^{2} \text{ kg}^{-2}$
Permeability of free space,	μ_0	=	$4\pi \times 10^{-7} \text{ H m}^{-1}$
Permittivity of free space,	ε_0	=	$8.85 \times 10^{-12} \text{ F m}^{-1}$
Planck constant,	h	=	6.63×10^{-34} J s
Proton rest mass,	m_p	=	1.67×10^{-27} kg
Rydberg constant,	R_∞	=	$1.0974 \times 10^{7} \text{ m}^{-1}$
Solar constant	S_o	=	$1.37 \times 10^{3} \text{ W m}^{-2}$
Sun's mass	$1M_\odot$	=	1.989×10^{30} kg
Stefan-Boltzmann constant	σ	=	$5.670 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
Velocity of light in vacuo,	c	=	$3.00 \times 10^{8} \text{ m s}^{-1}$
	1 eV	=	1.60×10^{-19} J

SECTION A

1. Answer **ALL** parts of the question:

(a) How many times as much light can be collected by one of the 10 metre aperture Keck telescopes compared to an average amateur telescope of aperture 20cm?

(3 marks)

Answer:
Light gathering power $\propto$ objective area.
For Keck; $\propto \pi \times 5^2$
For amateur telescope; $\propto \pi \times 0.1^2$
Ratio = $5^2/0.1^2$ = 2,500 times as much light.

(b) The theoretical resolving power R in radians of an astronomical telescope of aperture D at a wavelength λ is given by:

$$R = 1.22 \times \frac{\lambda}{D}.$$

What is the theoretical resolving power in arcsec of a 10 metre aperture Keck telescope for light of wavelength 500 nanometres? Give one reason why this resolution would never be achieved in practice.

(3 marks)

Answer:

$$R = 206265 \times 1.22 \times \frac{500 \times 10^{-9}}{10} = 2.5 \times 10^{-7}\, arc\mathrm{sec}$$

Atmospheric turbulence.

(c) At 02h on January 15th 2011 local sidereal time (LST for Salford is 09h 27m. The bright star Regulus has Right Ascension (RA); 10h 08m. Calculate Regulus' hour angle (HA) at this time and state whether Regulus is east or west of Salford's meridian.

(4 marks)

Answer:

HA (Regulus) = LST – RA = 09h 26m 55.2s – 10h 08m ≅ -41m;
i.e. = 24h – 41m = 23 19m.
Regulus is east of Salford's meridian.

(d) The polar form of the ellipse equation gives the radius vector *r* in terms of the semi major axis a, the eccentricity e and the true anomaly θ as:

$$r = \frac{a(1-e^2)}{1+e\cos\theta}.$$

Show that the periapsis distance r_1 is given by $r_1 = a(1-e)$. Given that the Earth' orbital eccentricity is 0.0167 use this to calculate the Earth's perihelion distance.

(4 marks)

Answer:

At periapsis, θ = 0; i.e. cos θ = 1 and so:

$$r_1 = \frac{a(1-e^2)}{1+e} = \frac{a(1+e)(1-e)}{1+e} = a(1-e)$$

r_1 = a(1 – e) = 1.496 × 10^{11} × (1 - 0.0167) = 1.471 × 10^{11} m

(e) The star Vega has an apparent magnitude m of exactly 0.0. Its parallax *p* is 1.289 ×10^{-1}arcsec. Calculate Vega's absolute magnitude *M*, given the distance modulus formula:

$$m – M = 5\text{Log } D – 5.$$

(4 marks)

Answer:

Distance D(parsec) = 1/p = 7.76.
Distance modulus formula:
m – M = 5LogD – 5
M = m – 5Log D + 5
= 0.0 – 5Log7.76 + 5 = 0.55.

(f) The bolometric magnitude scale is adjusted so that main sequence stars of spectral class F5 have a bolometric correction (BC) of 0.0. Explain very briefly why these stars are chosen.

(2 marks)

Answer:

Spectral class F5 stars emit most of their radiation in the visible part of the spectrum.

(g) The ionisation energy of the n = 2 energy level for hydrogen is 3.4eV. Calculate the wavelength of a photon which has this energy and state the name given to the important feature on stellar spectra which corresponds to this wavelength.

(4 marks)

Answer:

Energy; E= $3.4 \times 1.602 \times 10^{-19} = 5.45 \times 10^{-18}$joules.

Wavelength; λ = hc/E = $\frac{6.626\times10^{-34}\times3\times10^{8}}{5.45\times10^{-19}} = 3.647\times10^{-7}\,m$ i.e. 364.7 nanometres or 3647Å.

This corresponds to the Balmer jump or Balmer discontinuity.

(h) Explain in a few lines why forbidden transitions such as [OIII] λ5007 would never be observed in a physics laboratory but are very common in the spectra of objects such as planetary nebulae.

(3 marks)

Answer:

Conditions of density in a physics lab would result in collisional de-excitation long before the electron had a chance to undergo a forbidden downward transition. Large size plus extremely low density in a planetary nebula result in forbidden transitions taking place before collisional de-excitation can occur

(i) With the aid of symbols, list the three main stages of the proton-proton chain fusion reaction in stars.

(4 marks)

Answer:

$^{1}H + {}^{1}H \rightarrow {}^{2}H + \nu + e^{+}$
$^{1}H + {}^{2}H \rightarrow {}^{3}He + \gamma$
$^{3}He + {}^{3}He \rightarrow {}^{4}He + {}^{1}H + {}^{1}He$

(j) State the main observed difference between Population 1 and Population 2 stars and also state which are the older.

(3 marks)

Answer:

Spectra of pop 1 stars richer in metals or heavier elements. Pop 2 stars are older than pop 1 stars.

(k) The optical depth from the topmost layer of the Sun's photosphere down to a physical depth of400 km is approximately 3.5. Calculate the fraction of radiation emitted at this depth which reaches the photosphere's surface.

(3 marks)

Answer:

$$I = I_0 e^{-\tau}$$

$$\frac{I}{I_0} = e^{-3.5} = 0.03$$

(l) The red shift in the spectra of distant galaxies was originally interpreted as a Doppler shift. It is now referred to as a cosmological red shift. Explain very briefly the difference between the two.

(3 marks)

Answer:

A Doppler shift implies that the galaxy is moving through space. The cosmological red shift implies that spacetime is expanding; carrying the distant galaxies with it.

SECTION B

2. Answer **ALL** parts of the question:

(a) Given that the ratio of the flux received from two stars whose apparent magnitudes differ by exactly 5.0 is exactly 100.0, derive the distance modulus formula

$$m – M = 5\text{Log}(D) -5,$$

where m and M are the apparent and absolute magnitudes respectively for a star whose distance is D (pc).

(10 marks)

Answer:

Mag. Diference = 5.0 → flux ratio = 100

Mag. Difference = 1.0 → flux ratio = $\sqrt[1/5]{100} = 2.512$

Mag. Difference m – M → flux ratio $2.512^{(m-M)} = \frac{f_{10}}{f_D}$

f_D = flux at D parsec

f_{10} = flux at 10 parsec

$\frac{f_{10}}{f_D} = \frac{D^2}{100}$ Inverse square law for flux.

$2.512^{(m-M)} = \frac{D^2}{100}$

$(m-M)Log2.512 = 2LogD - 2 = 0.4(m-M)$

Hence:

$m - M = 5\log_{10} D - 5$

(b) The star Alnilam in the constellation of Orion has the following observational properties:

m = 1.7
B – V = -0.2
Parallax = 2.4 × 10^{-3}arcsec

(i) Calculate Alnilam's distance in pc.

(2 marks)

Answer:

Distance (pc) = $\frac{1}{parallax} = \frac{1}{2.4 \times 10^{-3}} = 416.7\,pc$

(ii) Use the distance modulus formula to calculate Alnilam's absolute magnitude.

(3 marks)

Answer:

M = m – 5LogD + 5
= 1.7 – 5Log416.7 + 5
= 1.7 – 13.1 + 5 = -6.4.

(iii) By comparing this to the Sun's absolute magnitude of +4.83, calculate Alnilam's luminosity in watts. (Assume all radiation is emitted in the visual part of the spectrum; *Note: the Sun's luminosity is 3.826×10^{26}watts).*

(8 marks)

Answer:

$$\frac{L_{\text{Alnilam}}}{L_{Sun}} = \frac{f_{\text{Alnilam}}}{f_{Sun}} = 2.512^{(M_{Sun}-M_{\text{Alnilam}})}$$

$$= 2.512^{(4.83--6.4)} = 2.512^{11.23} = 3.1\times10^{4}$$

$L_{Sun} = 3.826 \times 10^{26}$watts.

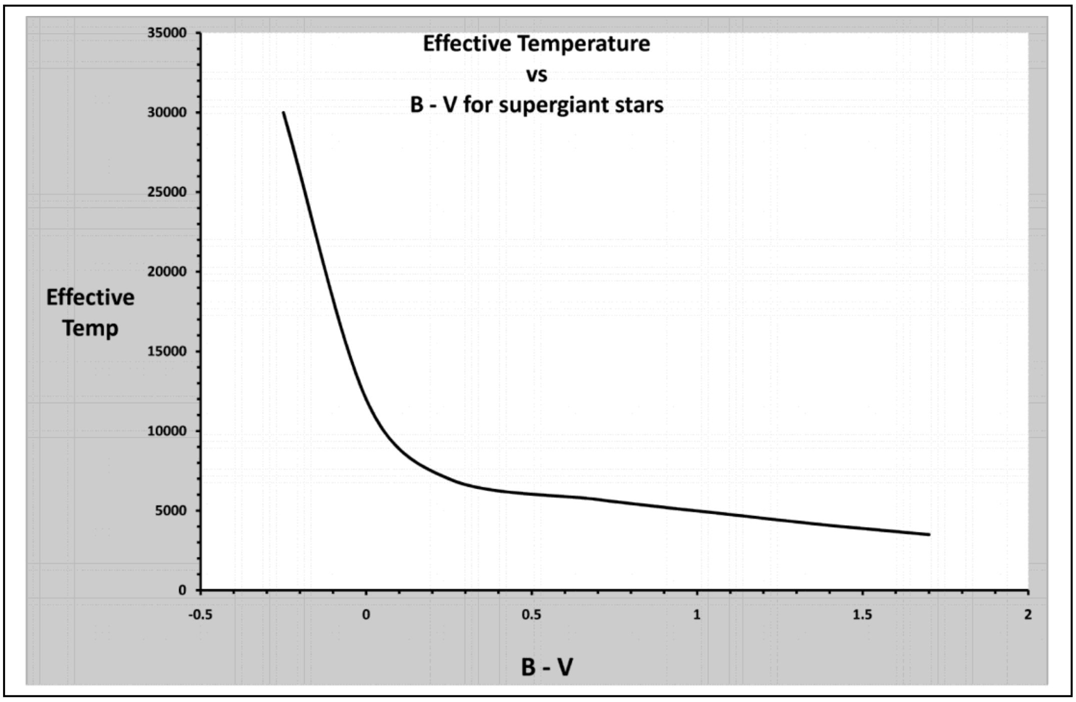
$\therefore$ $L_{\text{Alnilam}} = 3.1.\times 10^{4} \times 3.826 \times 10^{26} = 1.19 \times 10^{31}$watts.

(iv) The plot below shows the variation of $B - V$ colour index with effective temperature for supergiant stars like Alnilam. Estimate Alnilam's effective temperature and use the Stefan Boltzmann equation

$$F = \sigma T^{4} \text{ watts per square metre}$$

to estrimate Alnilam's diameter

(7 marks)



Answer:

From the plot, Alnilam's effective temp T $\cong$ 25000K.

Boltzmann:

$f = 5.67 \times 10^{-8} \times 25000^4 = 2.2 \times 10^{10}$ watts per sq m.

Surface area of Alnilam = $L_{Alnilam}/f$;
= $1.1 \times 10^{31}/2.2 \times 10^{10} = 5 \times 10^{20}$ sq m.

$$Diameter = 2\sqrt{\frac{5\times10^{20}}{4\pi}} = 1.26\times10^{10}$$ m or about 18 times that of the Sun.

3. Answer **ALL** parts of the question:

(a) State which spectral line broadening mechanism can be used to distinguish between giant and dwarf stars and explain very briefly how this is so.

(4 marks)

Answer:

Pressure or collisional broadening.

Giant stars tend to have less dense surface layers; so lines tend to be narrow. By contrast the higher density surface layers of dwarf stars often show evidence of pressure broadened lines.

(b) For a thermally broadened emission line the intensity f as a function of wavelength shift $\Delta\lambda$ within the line profile is given by

$$f(\Delta\lambda) = \frac{\lambda_0^2}{\sqrt{2\pi}c\Delta\lambda_{Dop}^2} e^{\frac{-(\Delta\lambda)^2}{2\Delta\lambda_{Dop}^2}},$$

where λ_0 is the line centre wavelength, *c* is the speed of light and $\Delta\lambda_{Dop}$ is the standard deviation or variance value of $\Delta\lambda$.

Show that $\Delta\lambda_{Dop} = 0.425 \times$ Full Width Half Maximum

where FWHM is the full width of the line profile at half maximum intensity.

(12 marks)

Answer:

At line centre;

$$f(0)=\frac{\lambda_0^2}{\sqrt{2\pi}c\Delta\lambda_{Dop}^2}$$

At half intensity;

$$\frac{\lambda_0^2}{2\sqrt{2\pi}c\Delta\lambda_{Dop}^2}=\frac{\lambda_0^2}{\sqrt{2\pi}c\Delta\lambda_{Dop}^2}e^{\frac{-(\Delta\lambda_{1/2})^2}{2\Delta\lambda_{Dop}^2}}$$

$$0.5=e^{\frac{-(\Delta\lambda_{1/2})^2}{2\Delta\lambda_{Dop}^2}}$$

Take natural logs of both sides;

$$Ln0.5=-Ln2=\frac{-(\Delta\lambda_{1/2})^2}{2\Delta\lambda_{Dop}^2}$$

$$\Delta\lambda_{1/2}=\sqrt{2Ln2}\Delta\lambda_{Dop}$$

$$FHWM=2\Delta\lambda_{1/2}=2\sqrt{2Ln2}\Delta\lambda_{Dop}$$

Hence;

$$\Delta\lambda_{Dop}=0.425\times FWHM$$

(c) The Doppler effect is described by the and the Doppler thermal velocity written in terms of the temperature *T* is given by

$$v_{Dop}=\sqrt{\frac{2kT}{m}}.$$

Calculate the temperature if the FWHM of the hydrogen alpha line (λ_0 = 656.3 nanometres) is 0.1 nanometres. Assume that the mass *m* of the hydrogen atom is $1,67 \times 10^{-27}$kg.

(10 marks)

Answer:

FWHM = 0.1 nanometre → $\Delta\lambda_{Dop}$ = 0.0425 nanometres = 4.25 × 10^{-11}m.

$$v_{Dop} = \frac{c\Delta\lambda_{Dop}}{\lambda_0} = \frac{3\times10^8 \times 4.25\times10^{-11}}{656.3\times10^{-9}} = 1.94\times10^4\, ms^{-1}$$

$$T = \frac{v_{Dop}^2 m}{2k} = \frac{3.76\times10^8 \times 1.67\times10^{-27}}{2\times1.38\times10^{-23}} = 22{,}750K$$

(d) If absorption lines from different elements are produced in different layers of a star's photosphere how will this affect the thermal broadening of lines from a given element?

(4 marks)

Answer:

Lines formed within deeper layers do so at higher temperatures & should be broader as a result.

4. Answer **ALL** parts of the question:

(a) The Doppler Effect can be used to determine the radial velocities of the two stars of a binary system. Explain briefly why it is necessary to observe eclipsing binary stars in order to determine the actual orbital velocities.

(3 marks)

Answer;

For two stars to mutually eclipse each other, the plane of the orbit must be approximately at 90° to the plane of the sky. This means that the maximum observed radial velocity is approximately equal to the orbital velocity.

(b) In a binary system the two component stars (of unknown masses M_1 and M_2) move in circular orbits of radii a_1 and a_2 respectively about their common centre of mass with orbital period P. Newton's law of gravity for the force between two bodies of mass M_1 and M_2 and separation a is given by

$$F = \frac{GM_1M_2}{a^2}$$

and the force on a body of mass M in a circular orbit of radius a moving with velocity v is given by

$$F = \frac{Mv^2}{a}$$

Derive Newton's version of Kepler's third law for circular orbits;

$$\frac{G(M_1 + M_2)}{a^3} = \frac{4\pi}{P^2}$$

(12 marks)

Answer;

Force between the two stars:

$$F = \frac{GM_1M_2}{a^2};$$

Also for circular orbits:

$$F = \frac{M_1v_1^2}{a_1} = \frac{M_2v_2^2}{a_2}$$

For orbital period P:

$$P = \frac{2\pi a_1}{v_1} = \frac{2\pi a_2}{v_2}$$

i.e.: $v_1^2 = \frac{4\pi^2 a_1^2}{P^2} and v_2^2 = \frac{4\pi^2 a_2^2}{P^2}$

Inserting these into the equations for F:

$$\frac{GM_1M_2}{a^2} = \frac{4\pi^2 M_1 a_1}{P^2} \Rightarrow \frac{GM_2}{a^2} = \frac{4\pi^2 a_1}{P^2}$$

And:

$$\frac{GM_1M_2}{a^2} = \frac{4\pi^2 M_2 a_2}{P^2} \Rightarrow \frac{GM_1}{a^2} = \frac{4\pi^2 a_2}{P^2}$$

Add these two together:

$$\frac{G(M_1 + M_2)}{a^3} = \frac{4\pi^2}{P^2}$$

Noting that $a = a_1 + a_2$.

(c) Two stars of mass M_1 and M_2 are observed to move in circular orbits at distances of $a_1 = 7.66 \times 10^9$m and $a_2 = 2.15\times10^{10}$m respectively from their common barycentre with an orbital period of 17.36 days.

Calculate the masses of the two stars in terms of the Sun's mass $1M_S$.
(Hint: $M_1a_1 = M_2a_2$).

(12 marks)

Answer:

$M_1a_1 = M_2a_2$

$$\frac{M_1}{M_2} = \frac{a_2}{a_1} = \frac{2.15\times10^{10}}{7.66\times10^{9}} = 2.8$$

$M_1 = 2.8M_2$.

$a = a_1 + a_2 = 2.92\times10^{10}$m.

$P^2 = 2.25\times10^{12}$s.

$$M_1 + M_2 = \frac{4\pi^2a^3}{GP^2} = \frac{12.57\times2.49\times10^{31}}{6.672\times10^{-11}\times2.25\times10^{12}} = 2.08\times10^{30}\,kg$$

$3.8M_2 = 2.08 \times 10^{30} \rightarrow M_2 = 5.47 \times 10^{29}$kg

$\rightarrow M_1 = 1.53 \times 10^{30}$kg,

i.e. $M_1 = 0.77M_S$; $M_2 = 0.275M_s$.

(d) An eclipsing binary with an orbital inclination of 90° consists of perfectly spherical stars of equal diameter. What important information is provided by the depths of the primary and secondary minima in the light curve?

(3 marks)

Answer:

The magnitudes of the individual stars.

5. Answer **ALL** parts of the question:

(a) Make a simple sketch of an edge on view of the Galaxy and indicate the location of:

(i) Population 1 & population 2 stars.

(2 marks)

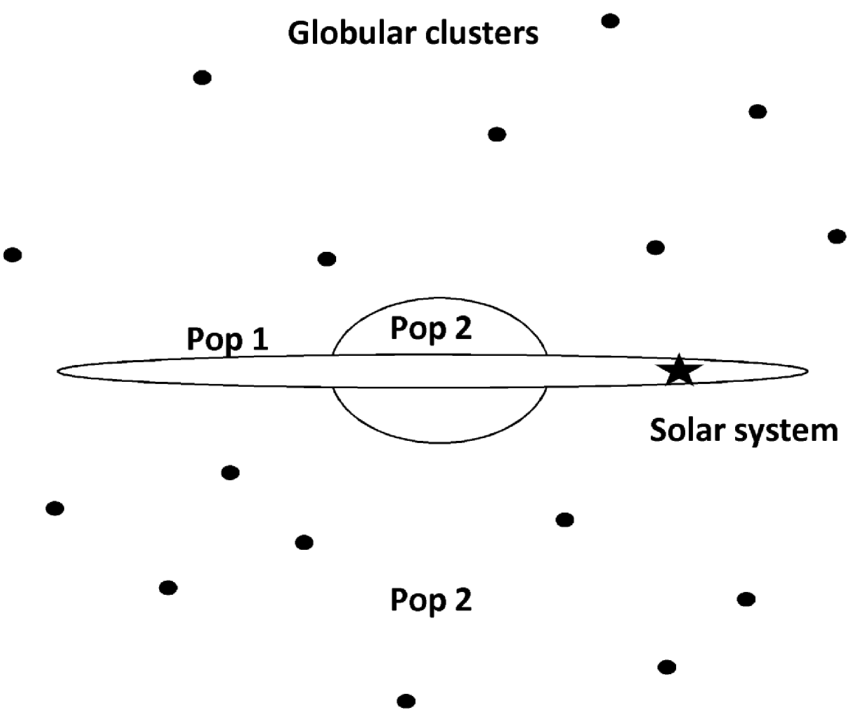
(ii) Globular clusters.

(2 marks)

(iii) The approximate location of the Solar System.

(2 marks)

Answer:



(b) Explain briefly in words what is meant by the local standard of rest for the Galaxy.

(4 marks)

Answer:

The vector sum is taken of the space velocity (speed & direction) of a suitable sample of solar neighbourhood stars. This then defines a frame of reference in which the Sun's space velocity is assumed to have the same magnitude but opposite direction; i.e. the local standard of rest is one in which the mean space velocity of local stars equals zero.

(c) By comparing the gravitational potential energy with the relative kinetic energy between two stars of mass *m*, separation *d* and relative velocity *v*, show the following:

(i) In the rest frame of one of the stars, the second star has an effective 'collision radius' given by:

$$d = \frac{2GM}{v^2}.$$

(5 marks)

Answer:

'Collision' occurs if Potential energy > kinetic energy:

$$\frac{Gm^2}{d} > \frac{mv^2}{2}$$

Or:

$$d < \frac{2Gm}{v^2}$$

This defines the 'collision radius.

(ii) If the collision cross sectional area of this star sweeps out a cylindrical volume of space whose volume equals 1 cubic parsec $(1\text{pc})^3$, show that the mean free path between collisions is given by

$$l = \frac{(pc)^3}{\pi\left(\frac{2Gm}{v^2}\right)^2 n}$$

Where *n* is the local number density of neighbouring stars which are assumed to be at rest.

(4 marks)

Answer:

Cylindrical volume = 1 cubic parsec; i.e.

$(pc)^3 = \pi\left(\frac{2Gm}{v^2}\right)^2 \times L$; Where L = length of cylinder.

If local star number density = n per cubic parsec, then mean free path:

$$l = \frac{L}{n} = \frac{(pc)^3}{\pi\left(\frac{2Gm}{v^2}\right) n}$$

(iii) Thus show that the mean time between collisions is given by

$$t = \frac{v^3 \times (pc)^3}{4\pi G^2 m^2 n}$$

(4 marks)

Answer:

$$t = \frac{l}{v}$$

i.e.

$$t = \frac{v^3 \times (pc)^3}{4\pi G^2 m^2 n}$$

(iv) Calculate the value of t in years for the solar neighbourhood by assuming that v = 20kms^{-1}; m = 1 solar mass and n = 1.
(Note: One parsec (1pc) = 3.086×10^{16}m).

(4 marks)

Answer:

$$t = \frac{(2\times10^4)^3 \times (3.086\times10^{16})^3}{4\pi \times (6.672\times10^{-11})^2 \times (1.989\times10^{30})^2 \times 1} = 1.086\times10^{21}\,\text{sec} = 3.36\times10^{13}\,\textit{years}.$$

(v) Comment on your answer.

(3 marks)

Answer:

This far exceeds the estimated age of the Galaxy of about 10^{10} years. Thus in all probability no stellar 'collisions' have ever occurred.