Coefficient of Restitution

David Harrison

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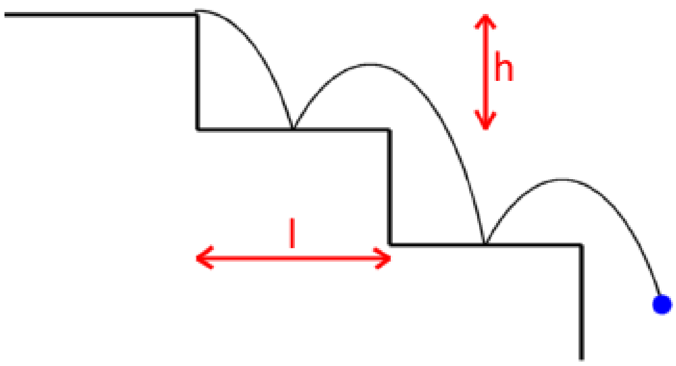
1 Introduction

A small ball is placed at the edge of the top step of a flight of steps. Each step has identical dimensions. The ball is kicked horizontally, perpendicular to the edge of the top step. On its first and second bounces it lands exactly in the middle of the first and second steps down from the top. Find the coefficient of restitution between the ball and the first step.

The ball continues bouncing down the steps hitting the middle of each successive step. What is the coefficient of restitution between the ball and the remaining steps?

2 Diagram

Below is a diagram of the path of the ball:



If the ball is said to take time t to travel a distance l horizontally, then before the first bounce only a time $\frac{t}{2}$ has elapsed. This gives an expression for h in terms of g and t:

$$s = ut + \frac{1}{2}at^2$$
$$-h = 0t - \frac{1}{2}g\left(\frac{t}{2}\right)^2$$
$$-h = -\frac{1}{2}g\frac{t^2}{4}$$
$$h = \frac{1}{8}gt^2$$

This relationship is extremely useful and will be used in all subsequent parts.

3 First Bounce

Immediately before the first bounce, the vertical component of the velocity of the ball is:

$$\text{s} = \text{vt} - \frac{1}{2}\text{at}^2$$

$$-\text{h} = \text{v}_0\frac{t}{2} + \frac{1}{2}\text{g}\left(\frac{t}{2}\right)^2$$

$$-\text{h} = \frac{1}{2}\text{v}_0\text{t} + \frac{1}{8}\text{gt}^2$$

$$\frac{1}{2}\text{v}_0\text{t} = -\frac{1}{8}\text{gt}^2 - \text{h}$$

$$\frac{1}{2}\text{v}_0\text{t} = -\frac{1}{8}\text{gt}^2 - \frac{1}{8}\text{gt}^2$$

$$\frac{1}{2}\text{v}_0\text{t} = -\frac{2}{8}\text{gt}^2$$

$$\text{v}_0\text{t} = -\frac{4}{8}\text{gt}^2$$

$$\text{v}_0 = -\frac{4}{8}\text{gt}$$

We can use the fact that the ball must go on to have a vertical displacement of -h in a further time t to calculate the velocity that the ball has immediately after the first bounce:

$$\text{s} = \text{ut} + \frac{1}{2}\text{at}^2$$

$$-\text{h} = \text{u}_0\text{t} - \frac{1}{2}\text{gt}^2$$

$$\text{u}_0\text{t} = \frac{1}{2}\text{gt}^2 - \text{h}$$

$$\text{u}_0\text{t} = \frac{1}{2}\text{gt}^2 - \frac{1}{8}\text{gt}^2$$

$$\text{u}_0\text{t} = \frac{3}{8}\text{gt}^2$$

$$\text{u}_0 = \frac{3}{8}\text{gt}$$

Now that expressions have been derived for the vertical component of the velocity of the ball before and after the first bounce, the coefficient of restitution for the first step can be calculated:

$$\text{COR}_0 = \left|\frac{u_0}{v_0}\right|$$

$$\text{COR}_0 = \left|\frac{\frac{3}{8}gt}{-\frac{4}{8}gt}\right|$$

$$\text{COR}_0 = \left|\frac{3}{-4}\right|$$

$$\text{COR}_0 = 0.75$$

4 Second Bounce

Using the same information that allowed calculation of u_0, the vertical component of the velocity just before the second bounce can be calculated:

$$s = vt - \frac{1}{2}at^2$$

$$-h = v_1t + \frac{1}{2}gt^2$$

$$v_1t = -\frac{1}{2}gt^2 - h$$

$$v_1t = -\frac{1}{2}gt^2 - \frac{1}{8}gt^2$$

$$v_1t = -\frac{5}{8}gt^2$$

$$v_1 = -\frac{5}{8}gt$$

There is no need to calculate u_1 since it is the same as u_0, and therefore the coefficient of restitution for the second step (and all other steps) can be calculated now:

$$COR_1 = \left|\frac{u_1}{v_1}\right|$$

$$COR_1 = \left|\frac{\frac{3}{8}gt}{-\frac{5}{8}gt}\right|$$

$$COR_1 = \left|\frac{3}{-5}\right|$$

$$COR_1 = 0.6$$

5 Conclusion

It is rather surprising that with so little information given in the problem, exact numerical solutions can be found.

The coefficient of restitution between the ball and the first step is 0.75.

The coefficient of restitution between the ball and all other steps is 0.6.