

Frosty the snowman is made from two uniform spherical snowballs, of radii $2R$ and $3R$. The smaller (which is his head) stands on top the larger. As each snowball melts, its volume decreases at a rate which is directly proportional to its surface area. The constant of proportionality being the same for each snowball. During melting, the snowballs remain spherical and uniform.

When frosty is half his initial height, show that the ratio of his volume to his initial volume is $37 : 224$. Let V and h denote Frosty's total volume and height, respectively, at time t . Show that, for $2R < h \leq 10R$:

$$\frac{dV}{dh} = \frac{\pi}{8} (h^2 + 4R^2)$$

And derive the corresponding expression for $0 \leq h < 2R$. Sketch $\frac{dV}{dh}$ as a function of h , for $4R \geq h \geq 0$, hence give a rough sketch of V as a function of h .