Chocolate Orange

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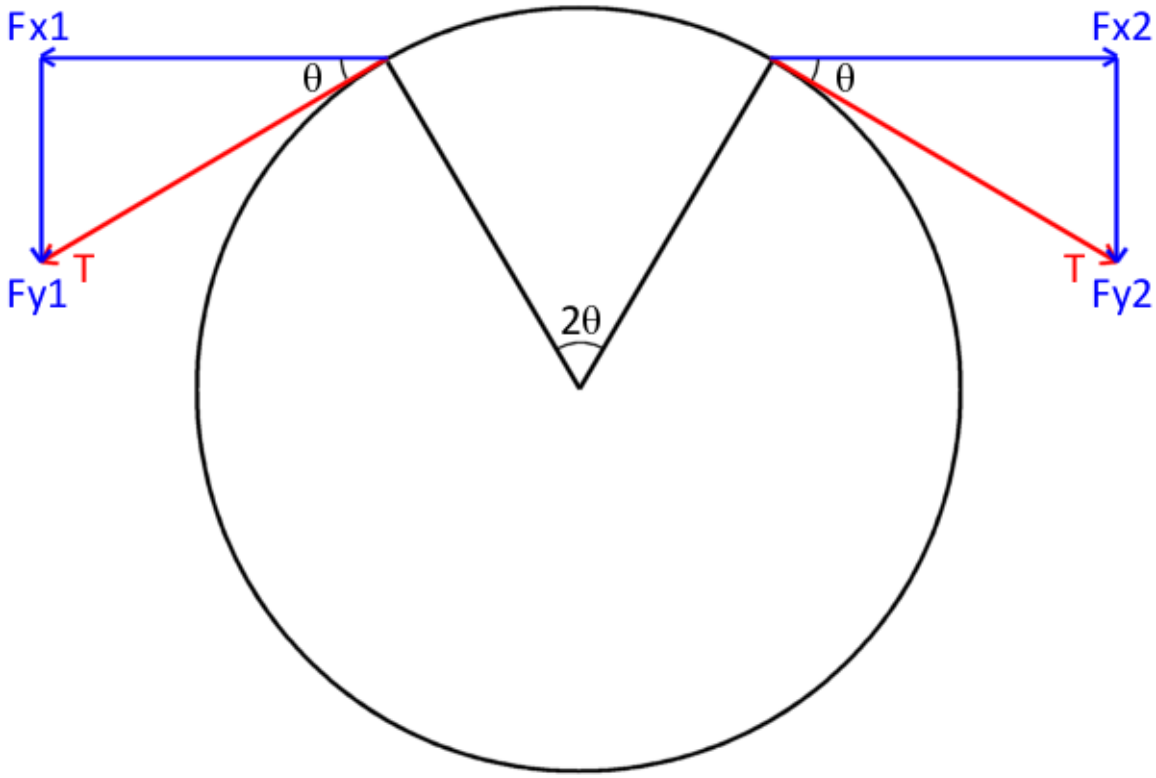
1 Introduction

A chocolate orange is a sphere of delicious smooth uniform chocolate of mass M and radius a, sliced into segments by planes through a fixed axis. It stands on a horizontal table with this axis vertical and it is held together by a narrow ribbon round its equator. Show that the tension in the ribbon is at least $\frac{3}{32}$Mg.

(You may assume that the centre of mass of a segment of angle 2θ is at distance $\frac{3\pi a sin\theta}{16\theta}$from the axis.)

2 Determining Net Force on Segment From Tension

Below is a diagram showing the orange from above, marked out is a segment of angle 2θ.



The tension acting on the segment due to the ribbon is shown in red on either side and has been resolved into component forces.

It is clear that F_{x1} and F_{x2} are of the same magnitude but opposite direction, therefore they contribute no net force to the segment and can be ignored. Similarly it can be seen that F_{y1} and F_{y2} are of the same magnitude and the same direction, therefore the net force on the segment can be easily determined in terms of θ:

$$sin\theta = \frac{F_{y1}}{T}$$

$$\therefore F_{y1} = Tsin\theta$$

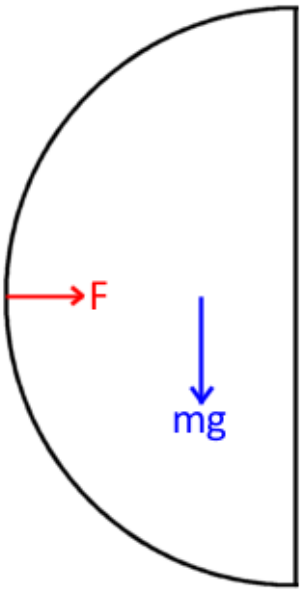
And since:
$F_{y2} = F_{y1}$

$$F = 2Tsin\theta$$

This force acts directly towards the centre of the orange.

3 Taking Moments

Below is a diagram of a single segment looked at edge on:



If moments are taken about the lower right corner of this segment, then to keep the segment in place, the clockwise moment of the horizontal force, F, must be greater than or equal to the anti-clockwise moment of the weight, mg.

For a segment of angle 2θ:

$F = 2Tsin\theta$

$m = M*\frac{2\theta}{2\pi}$
$m = M*\frac{\theta}{\pi}$

Now taking moments about the lower right corner:

$a * 2Tsin\theta \geq \frac{3\pi asin\theta}{16\theta} * Mg * \frac{\theta}{\pi}$

$a * 2Tsin\theta \geq \frac{3asin\theta}{16} * Mg$

$2Tsin\theta \geq \frac{3sin\theta}{16} * Mg$

$2T \geq \frac{3}{16} * Mg$

$T \geq \frac{3}{32} * Mg$

Therefore it has been shown that to hold the orange together, the tension in the ribbon must be at least $\frac{3}{32}$Mg.