Refraction at a Spherical Surface

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1 Introduction

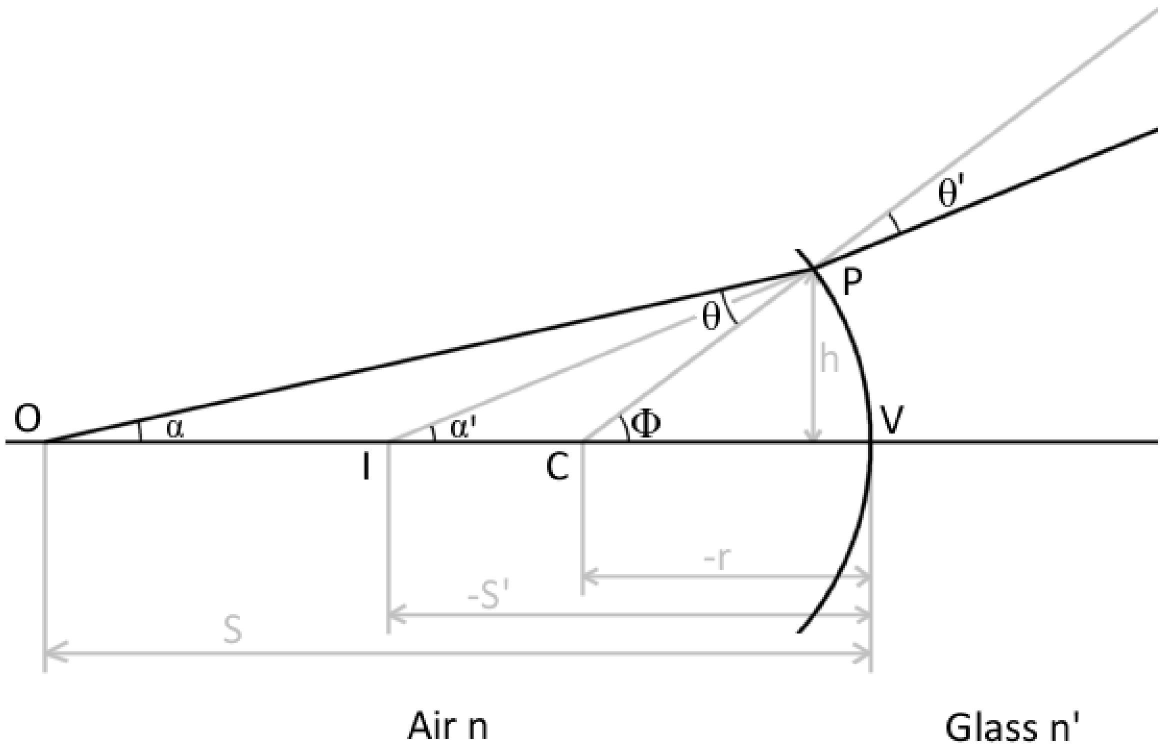
For refraction at a concave spherical surface with radius of curvature r, show that:

$$\frac{n}{S}+\frac{n'}{S'}=\frac{n'-n}{r}$$

Where n and n' are refractive indices of the media at either side of the interface, and S and S' are the object and image distances respectively. Small angle approximation is assumed to be valid.

2 Derivation

Below is a diagram of the interface:



Light leaves point O and is refracted at point P. There is a virtual image formed at point I and the centre point of the curvature of the lens is at C. Due to the sign convention used, the distance to the image is -S' and the distance to the centre of curvature is -r, both measured from point V.

From triangle COP:

$$\alpha + \theta + 180 - \Phi = 180$$
$$\therefore \theta = \Phi - \alpha$$

From triangle IOP:

$$\alpha' + \theta' + 180 - \Phi = 180$$
$$\therefore \theta' = \Phi - \alpha'$$

Also, using the small angle approximation:

$$\Phi = \frac{h}{-r} = -\frac{h}{r}$$
$$\alpha = \frac{h}{s}$$
$$\alpha' = \frac{h}{-S'} = -\frac{h}{S'}$$

Now using Snell's law and the small angle approximation:

$$n sin\theta = n' sin\theta'$$

$$n\theta = n'\theta'$$

$$n(-\frac{h}{r} - \frac{h}{S}) = n'(-\frac{h}{r} + \frac{h}{S'})$$

$$n(\frac{h}{r} + \frac{h}{S}) = n'(\frac{h}{r} - \frac{h}{S'})$$

$$\frac{n}{r} + \frac{n}{S} = \frac{n'}{r} - \frac{n'}{S'}$$

$$\frac{n}{S} + \frac{n'}{S'} = \frac{n'}{r} - \frac{n}{r}$$

$$\frac{n}{S} + \frac{n'}{S'} = \frac{n' - n}{r}$$

Therefore it has been shown that the formula holds for a concave interface, as well as a convex interface.